NICHOLAS J. DARAS

CURRICULUM VITAE and ANNOTATED SCIENTIFIC ACTIVITIES

2022

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CURRICULUM VITAE

Nicholas J. Daras
Professor of Mathematics
Dean of the Hellenic Military Academy (2016-2020)[1]

http://www.sse-tuc.edu.gr/daras/Daras_CV.pdf

Personal Data.

Date of Birth: June 6, 1961
Marital Status: Married with one child
Nationality: Hellenic
Sex: Male

Current Work Address: Department of Mathematics and Engineering Sciences
Faculty of Military Sciences
Hellenic Military Academy, 166 73, Vari Attikis, Greece

Home Address: Jean Moreas 19, 152 32 Chalandri, Athens, Greece

Phone numbers: 0030 210 890 4332 / 0030 210 890 4217 (office)
0030 210 897 0232 (office FAX)
0030 210 689 3552 (home)
0030 693799 1649 (mobile)

E-mail addresses:njdaras@gmail.com , ndaras@sse.gr ,

Research Areas.

- Numerical Analysis (*Rational approximation, rational interpolation, best rational approximation and interpolation theory, rational approximation to vectors of mutually irrational numbers, orthogonal polynomials in numerical reconstruction of signals, Markov-type inequalities in multivariate complex approximation,acceleration of computational schemes, interpolation methods for the numerical evaluation of finite Baire measures, multidimensional logarithmic residue formulasfor solving systems of non linear equations, complex extrapolated successive overrelaxation,numerical solving of differential equationsand integral equations, numerical evaluation of integrals and derivatives, numerical optimization, Markov-type inequalities with applications in multivariate approximation*)
- Complex Analysis and Pure Mathematics (*Universal power series in one and several variables, summability transforms and analytic continuation in C^n, biholomorhic maps in C^n, integral representations in C^n, contact geometry, topological and differential calculus, complex analysis,mathematical analysis, geometric analysis, projectiveanddescriptive geometry*)
- History of Mathematics (*History of continued fractions, connection to rational approximation theory, history of deterministicmathematical combat theories*)
- Stochastic Modeling and Numerical Simulation in Operations Research (*Stochastic and renewalcombatmodels, reliability of military operations, missile allocation strategies and target coverage, numerical multi-agent simulation (in small to medium scale), strategic defense*)

[1] The Hellenic MilitaryAcademyhas been evaluated andlistedamong the ten (10) most prestigious military Academies in the world(seehttp://www.onlinecollege.org/10-most-prestigious-military-academies-in-the-world)

- **Quantum Computation & Cryptography** (*Topological quantum computations, biholomorphic codes and cryptosystems*)
- **Security** (*Security and formation of network-centric operations, information security, stochastic analysis of cyber attacks, epidemiological diffusion and discrete branching models for malware propagation in computer networks*)
- **Mathematical Modeling** (*Mathematical modeling of cyberspace and cyber secusrity, mathematical models in deterministic combattheory, mathematical models in portfolio analysis of defense, mathematical models in economy, mathematical models in fluid mechanics, mathematical models in military logistics*)
- **Big Data** (*Selective properties of big data*)
- **Data Management** (*Subjective preferences in data management*)
- **Prediction of Systemic Events** (*Prediction of peculiar systemic incidents, prediction of geopolitical events*)
- **Diophantine Equations** (*Beal' s Conjecture, Generalized Fermat Equation*).

Future Research Interests: *Visualization of complex functions, numerical simulation to stochastic differential equations, detection of initial points for the numerical solving of algebraic systems, sparse solutions of underdetermined linear systems and compressed sensing, unconventional computing (cellular automata, complexity, micromechanical encryption, computing in nanomachines), topological quantum computation, optimization of large quantum circuits, numerical representations in Biometrics, numerical constructing of universal series, universal series in* $\mathbb{C}^n$*, holomorphic mappings with respect to various topologies in* $\mathbb{C}^n$*, mathematical models in portfolio analysis*

Heteroreferences (until 28.6.2022)**> 230 mentions in papers** (Academia.edu, https://www.academia.edu/)

Graduate Education: *Thèse de Doctorat en Mathématiques*, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France, 1988, Thesis directed by Claude Brezinski. Thesis Examination Committee: *Gerard Cœuré* (President), *Jean Jacques Lœb* (Rapporteur), *Michael Eiermann* (Rapporteur) and *Claude Brezinski* (Examinateur).

Diplôme d' Etudes Approfondies (D.E.A.) des Mathématiques Pures, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France, 1985

Undergraduate Master's Degree: MSc in Mathematics, Department of Mathematics, National and Kapodistrian University of Athens, Athens, Greece, 1984

Honors, Distinctions: Award PhD Degrees with the highest distinction "TRES HONORABLE" of French Universities, 1988

Award of one of the awards given by the Academy of Athens for scientific publications in mathematics, 2002

Four (4) scientific publications appear on the website of the Department of Mathematics at California State University Fullerton with significant bibliography in different areas of mathematics, from the early 20th century until today (see

http://math.fullerton.edu/mathews/n2003/pade/PadeApproximationBib/Links/PadeApproximationBib_lnk_2.html

http://math.fullerton.edu/mathews/n2003/pade/PadeApproximationBib/Links/PadeApproximationBib_lnk_3.html http://math.fullerton.edu/mathews/c2003/TaylorSeriesBib/Links/TaylorSeriesBib_lnk_2.html

http://math.fullerton.edu/mathews/c2003/TaylorSeriesBib/Links/TaylorSeriesBib_lnk_3.html

http://math.fullerton.edu/mathews/c2003/riemannmapping/riemannmappingtheorembib/links/riemannmappingtheorembib_lnk_2.html)

Vice president of the Hellenic Socciety of Operations Research (EEEE) (2011-2013) (http://www.eeee.org.gr/PublicPages/a_0_el.aspx)

Scholarship: Fellowship for Graduate Studies by the Board of the Secretariat of the universities of Paris, 1984

Academic Positions Held: Assistant Professor (PresidentialDecree 407/80), Department of Mathematics, University of Aegean, Samos, Greece, 1990-1992

Lecturer in Mathematics, Hellenic Military Academy, Athens, Greece, 1992-2005

Professor in Mathematics and Statistics, as part of the curriculum in Economics and Management from the University Paris-Nord (Paris 13), set up to the InstituteofFrench Studiesin Athens, Greece, 1996-1999

Professor ofMathematics withprivate law contracts,Air Force Academy, 1994-2012

Professoron contract, Department of Sciences, School ofApplied Technology, Technological Educational Instituteof Halkida, 2001-2004

Unsalaried Professorof graduatecourses, Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, Athens, Greece, 2001-2010

Assistant Professor, Department of Mathematics and Engineering Sciences, Faculty of Military Science, Hellenic Military Academy, Athens, Greece, 2005-20/1/2010

AssociateProfessor, Department of Mathematics and Engineering Sciences, Faculty of Military Science, Hellenic Military Academy, Athens, Greece, 20/1/2010-18/2/2010

Full Professor, Department of Mathematics and Engineering Sciences, Faculty of Military Science, Hellenic Military Academy, Athens, Greece, 18/2/2010-present

Teaching: Teaching the course "*Numerical Analysis*" (1time) in the Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, Athens, Greece, 2001-2002

Introduction and teaching the course "*Numerical Optimization*" (12 times) to graduate students in the Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, Athens, Greece, 1999-2011

Teaching the course of "*Complex Analysis and Numerical Analysis*" (1 time) in the Department of Mathematics, University of Aegean, Samos, Greece, 1990-1991

Teaching the course "*Numerical Analysis*" (1time) in the Department of NaturalSciences, School of Applied Technology, Technological Educational Institute of Halkida, 2002-2003

Teaching the course "*Applied Mathematics (Numerical Analysis)*"(2times) in the Departmentof Aeronautical Sciences, Hellenic Air Force Academy, Dekelia, Greece,1998-2000

Teaching the course "*Complex Analysis of Several Variables*"(1time) in the Department of Mathematics, University of Aegean, Samos, Greece, 1990-1991

Teaching the course "*Mathematics*" (16times) in the Departmentof Aeronautical Sciences, Hellenic Air Force Academy, Dekelia, Greece, 1994-2000 and 2001-2011

Teaching the course "*Statistics*" (3 times) in the curriculum in Economics and Management from the University Paris-Nord (Paris 13), InstituteofFrench Studiesin Athens, Greece, 1996-1999

Teaching the course "*Mathematics*" (3 times) in the curriculum in Economics and Management from the University Paris-Nord (Paris 13), Institute of French Studies in Athens, Greece, 1996-1999

Teaching the course "*Applied Mathematics*" (1time) in the Department of Natural Sciences, School ofApplied Technology, Technological Educational Instituteof Halkida, 2002-2003

Teaching the course "*Mathematics I*" (3 times) in the Department of NaturalSciences, School ofApplied Technology, Technological Educational Instituteof Halkida, 2001-2004

Teaching the course "*Differential and Integral Calculus*" (1 time) in the Department of NaturalSciences, School ofApplied Technology, Technological Educational Instituteof Halkida, 2003-2004

Teaching the course "*Operations Research*" (1time) in the Department of Aeronautical Sciences, Hellenic Air Force Academy, Dekelia, Greece, 1998-1999

Introduction andteaching the course "*Operations Research and Military Applications*" (10 times) in the Faculty of Military Sciences of the Hellenic Military Academy, Vari, Greece, 2007-present

Introduction andteaching the course "*Differential Equations and Mathematical Theories of War*" (9 times) in the Faculty of Military Sciences of the Hellenic Military Academy, Vari, Greece, 1999-2002 and 2004-2010

Introduction andteaching the course "*Target Coverage and Target Analysis*"(9times) in the Faculty of Military Sciences of the Hellenic Military Academy, Vari, Greece, 2008-present

Introduction andteaching the course "*Missile Allocation Strategies*" (9 times) in the Faculty of Military Sciences of the Hellenic Military Academy, Vari, Greece, 2008-present

Introduction and teaching the course "*Search and Screening*" (9times) in the Faculty of Military Sciences of the Hellenic Military Academy, Vari, Greece, 2008-present

Introduction and teaching the course "*Chaos and Complexity, with Applications to the Artificial War*" (1 time) in the Faculty of Military Science of the Hellenic Military Academy, Vari, Greece, 2012

Teaching the course of "*Descriptive Geometry*" (16 times) in the Departmentof Aeronautical Sciences, Hellenic Air Force Academy, Dekelia, Greece, 1994-2000 and 2001-2011

Introduction and teaching of the course "*Mathematical Programming and Optimization*" (5 times) graduate students of the Postgraduate Program "APPLIED OPERATIONAL RESEARCH AND ANALYSIS" of the Hellenic Military Academy in collaboration with the Tecnical University of Crete, 2015-2019

Introduction and teaching of the course "*Special Topics in Operations Research*" (5 times) graduate students of the Postgraduate Program "APPLIED OPERATIONAL RESEARCH AND ANALYSIS" of the Hellenic Military Academy in collaboration with the Tecnical University of Crete, 2015-2019

Adninistrative Work:*Dean* of the Hellenic Military Academy (September 2016-present)

Director of the Department of Mathematics and Engineering Sciences in the Faculty of the Military Sciences of the Hellenic Military Academy (Febryary 2011-August 2016)

Organizer and *Director* of the Program of Postgraduate Studies being made the Hellenic Military Academy in collaboration with the Technical University of Crete in the object "APPLIED OPERATIONS RESEARCH & ANALYSIS" (2014-2019)

Organizer and *Deputy Director* of the Program of Postgraduate Studies being made the Hellenic Military Academy in collaboration with the Technical University of Crete in the object "SYSTEMS ENGINEERING" (2014-2019)

Organizer and *Director* of the Program of Postgraduate Studies being made the Hellenic Military Academy in collaboration with the Technical University of Crete in the object "OPERATIONS RESEARCH & DECISION MAKING" (2019-present)

Organizer and *Deputy Director* of the Program of Postgraduate Studies being made the Hellenic Military Academy in collaboration with the Technical University of Crete in the object "INTELLIGENT SYSTEMS " (2019-present)

Ten (10) Academics who could give information about me and my work if requested:

- **Claude Brezinski** (*Professeur Émérite d'Analyse Numerique à l'Université des Sciences et Technologies de Lille, France*), http://math.univ-lille1.fr/~brezinsk/
  E-mail address:claude.brezinski@univ-lille1.fr
- **Preda V. Mihăilescu** (*Professor at the Georg-August-University of Goettingen, Germany*), http://en.wikipedia.org/wiki/Preda_Mih%C4%83ilescu
  E-mail address:preda@uni-math.gwdg.de
- **Vassilios A. Dougalis** (*Emeritus Professor of Numerical Analysis at the University of Athens, Greece; FORTH, Institute of Applied and Computational Mathematics*), http://www.forth.gr/index_main.php?c=44&l=e
  E-mail address:dougalis@iacm.forth.gr
- **Nikolaos M. Missirlis** (*Professor of Numerical Analysis at the University of Athens, Greece*), http://parallel.di.uoa.gr/PSCL/N.Missirlis/
  E-mail address:nmis@di.uoa.gr
- **Panos Pardalos** (*Distinguished Professor of Industrial and Systems Engineering at the University of Florida, U.S.A.*), http://www.ise.ufl.edu/pardalos/
  E-mail address:panos.pardalos@gmail.com
- **Michael N. Vrahatis** (*Professor of Numerical Analysis at the University of Patras, Greece*), http://www.math.upatras.gr/~vrahatis/
  E-mail address:vrahatis@math.upatras.gr
- **Nikolaos Limnios** (*Professeur à l'Université de Technologie de Compiègne, France*),http://www.utc.fr/~nlimnios/
  E-mail address:Nikolaos.Limnios@utc.fr
- **Vassili Nestoridis** (*Emeritus Professor of Complex Analysis at the University of Athens, Greece*), http://www.math.uoa.gr/web/english/faculty/analysis/nestor.htm
  E-mail address:vnestor@math.uoa.gr
- **Michela Redivo Zaglia** (*Professor of Numerical Analysis at the University of Padova, Italy*), http://www.math.unipd.it/~michela/indexeng.html
  E-mail address:Michela.RedivoZaglia@unipd.it
- **Themistocles M. Rassias** (*Emeritus Professor at the National Technical University of Athens, Greece*), http://en.wikipedia.org/wiki/Themistocles_M._Rassias
  E-mail address:trassias@math.ntua.gr

Reviewer of Mathematical Reviews

Referee of several international mathematical Journals and Proceedings, including:

- Journal of Computational and Applied Mathematics
- Symbolic-Numerical Computing
- Journal of Applied Mathematics and Bioinformatics
- Journal of Computations and Modelling

Knowledge of Programming Languages-Software Packages-Operating Systems:

Fortran, C, C++, Pascal, Java, LaTeX, SPSS, S-Plus, R, Monitab, Amos (SPSS), Eviews, Statgraphics, Mathematica, MATLAB, Scilab

SCIENTIFIC ACTIVITIES

Nicholas J. Daras

Conferences Organization: Organising and chairing the national Conference on "*Missile Allocation Strategies and Multi-Agent Based Simulation of Combat*", held in the Hellenic Military Academy, November 2008

Organizing and chairing the international Conference on "*Applications of Mathematics and Informatics in Military Science (AMIMS)*", held in the Hellenic Military Academy, April 2011
(http://www.amazon.ca/books/dp/1461441080 ,
http://www.abebooks.com/products/isbn/9781461441083/8639264438 , and
http://www.buchpark.de/Applications_of_Mathematics_and_Informatics_in_Military_Science_9781461441083.html)

Organizing and chairing the International Conference on "*Cryptography and its Applications in Armed Forces*", held in the Hellenic Military Academy, April 2012
(http://www.sse.gr/files/cryptography%20conference-en.pdf)

Organizing and chairing the 2nd International Conference on "*Applications of Mathematics and Informatics in Military Science*", held in the Hellenic Military Academy, April 2013
(http://www.sse.gr/files/programma%20synedrioy.pdf and
http://www.sse.gr/files/Abstracts_con2.pdf)

Organizing and chairing the 2nd International Conference on "*Cryptography, Network Security and its Applications in Armed Forces*", held in the Hellenic Military Academy, April 2014
(http://www.sse.gr/files/2nd_CryptAAF_EN.pdf ; also
http://www.athensconventionbureau.gr/en/content/2nd-international-conference-cryptography-network-security-and-applications-armed-forces)

Organizing and chairing (with the Director of Artillery Directorate of the Hellenic Army, General Staff) the 3rd International Conference on "*Technology Trends and Scientific Applications in Artillery and other Military Science*", held in the Hellenic Artillery Shcool, May 2015
(http://www.army.gr/default.php?pname=Article&art_id=89691&cat_id=14&la=2)

Organizing and chairing the 3rd International Conference on "*Cryptography, Cyber- Security and Information Warfare*", held in the Hellenic Military Academy, May 2016
(http://sse.army.gr/sites/sse.army.gr/files/attachments/conference_announcement_in_english.pdf; also
http://sse.army.gr/sites/sse.army.gr/files/attachments/1.afissa_synedrioy.pdf)

Organizing and chairing the 4th International Conference on "Cryptography, Security and Information Systems", Hellenic Military Academy, 21th - 22th May 2020
(https://sse.army.gr/sites/sse.army.gr/files/attachments/conference_announcement.pdf)

Lectures Given: CONTINUOUS SEMINAR OF MATHEMATICS

Lecture title: *«Sur les fonctions spectrales pour z dans H(δ) et la pseudoconvexité»*Department of Mathematics, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France
January 1985

CONTINUOUS SEMINAR OF MATHEMATICS
Lecture title: *«Théorie spectrale et problème de Levi»*
Department of Mathematics, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France
February 1985

CONTINUOUS SEMINAR OF MATHEMATICS
Lecture title: *«Fonctions holomorphes à croissance bornée non-prolongeables dans les domaines de Reinhardt»*
Department of Mathematics, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France
June 1985

INVITED LECTURE GIVEN
Lecture title: *«L'algèbre extérieure d'un espace symplectique et formes différentielles sur* $J^1(M)$*»*
Department of Mathematics, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France
June 1988

INVITED LECTURE GIVEN
Lecture title: *«Opérateurs différentiels non-linéaires»*
Department of Mathematics, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France
June 1990

INVITED LECTURE GIVEN
Lecture title: *«Biholomorphic mappings in several complex variables»*
Department of Mathematics, AristotleUniversity of Thessaloniki, Thessaloniki, Greece
May 1991

INVITED LECTURE GIVEN
Lecture title: *«On Riemann's conformal mapping theorem in several complex variables»*
Department of Mathematics, National and Kapodistrian University of Athens, Greece
November 1992

INVITED LECTURE GIVEN
Lecture title: *«Positiveclosedcurrents in Complexmanifolds: structure and expansion»*
Department of Mathematics, National and Kapodistrian University of Athens, Greece
April 1993

INVITED LECTURE GIVEN
Lecture title: *«Accélération de la convergence de la distance Euclidienne et fonctions holomorphes à croissance bornée dans les domaines de Runge»*
Department of Mathematics, University PAUL SABATIER, Toulouse, France
June 1990

INTERNATIONAL CONFERENCE ON NUMERICAL ALGORITHMS,
Lecture title: *«Asymptotic methods for numerical sequences and growth conditions for holomorphic functions»*
Marrakech, Morocco
October 2001

INVITED LECTURE GIVEN
Lecture title: *« Interpolation methods for signal pointwise estimation and Padé-type approximants »*
Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, Greece
April 2003

1st INTERNATIONAL CONFERENCE ON APPLICATIONS OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES
Lecture title: «*Mathematicalmodelsfororganizingstrategicdefense: Layereddefense, theatremissiledefense, antiaircraftdefense*»
Hellenic Military Academy, Greece
April 2011

1st CONFERENCE OF THE HELLENIC MATHEMATICAL SOCIETY AND THE HELLENIC SOCIETY OF OPERATIONS RESEARCH,
Lecture title: «*Mathematicalmodelsfororganizingstrategicdefense: Layereddefense, theatremissiledefense, antiaircraftdefense*»
Conference Center of Technological Educational Institute of Piraeus, Greece
June 2011

28oHELLENIC CONFERENCE ON MATHEMATICAL EDUCATION. MATHEMATICAL MODELING: APPLICATIONS IN SCIENCE, TECHNOLOGY AND EDUCATION
Lecture title: *«Missile allocation strategies for a group of identical targets»*
Department Mathematics, National and Kapodistrian University of Athens, Greece
November 2011

NUMAN2012. FIFTH CONFERENCE IN NUMERICAL ANALYSIS.
Lecture title: *«A Direct Numerical Algorithm for Solving Systems of Non Linear Equations using Multidimensional Logarithmic Residue Formulas» (with Dimitrios Triantafyllou)*
Department Mathematics, University of Ioannina, Greece
September 2012

29oHELLENIC CONFERENCE ON MATHEMATICAL EDUCATION. MATHEMATICS: THEORY-PRACTICE-EXTENSIONS
Lecture title: *«Optimal placement of defense forces »(with Dimitrios Triantafyllou)*
Kalamata, Greece
November 2012

2nd INTERNATIONAL CONFERENCE ON APPLICATION OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES
Lecture title: «*Mathematical models of strategic defense: Layered defense and theater of missile defense*»
Hellenic Military Academy, Greece
April 2013

2nd INTERNATIONAL CONFERENCE ON APPLICATION OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES
Lecture title: «*Stochastic and renewal analysis of irregular warfare. Mathematical models and evaluation* »
Hellenic Military Academy, Greece
April 2013

2nd INTERNATIONAL CONFERENCE ON APPLICATION OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES

Lecture title: «*Military applications of portfolio analysis: A literature review*» (with P.Xidonas, N. J. Daras, George Mavrotas and J. Psarras)
Hellenic Military Academy, Greece
April 2013

2nd INTERNATIONAL CONFERENCE ON APPLICATION OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES
Lecture title: «*Numerical solution of the defense force optimal positioning problem*» (with D. Triantafyllou)
Hellenic Military Academy, Greece
April 2013

2nd INTERNATIONAL CONFERENCE ON APPLICATION OF MATHEMATICS AND INFORMATICS IN MILITARY SCIENCES
Lecture title: «*Reliability of military operations and systems*»
Hellenic Military Academy, Greece
April 2013

2nd INTERNATIONAL CONFERENCE ON CRYPTOGRAPHY, NETWORK SECURITY AND APPLICATIONS IN THE ARMED FORCES
Lecture title: «*Security and Formation of Network-Centric Operations*»
Hellenic Military Academy, Greece
April 2014

3rd INTERNATIONAL CONFERENCE ON TECHNOLOGY TRENDS AND SCIENTIFIC APPLICATIONS IN ARTILLERY AND OTHER MILITARY SCIENCES (TTSAAMS) (http://www.army.gr/default.php?pname=Article&art_id=89691&cat_id=14&la=2)
Lecture title: «*Systemic Geopolitical Modeling: Prediction of Geopolitical Events*»
Hellenic Artillery School, Greece
May 2015

3rd INTERNATIONAL CONFERENCE ON TECHNOLOGY TRENDS AND SCIENTIFIC APPLICATIONS IN ARTILLERY AND OTHER MILITARY SCIENCES (TTSAAMS) (http://www.army.gr/default.php?pname=Article&art_id=89691&cat_id=14&la=2)
Lecture title: «*Ellipsoid Targeting with Overlap*»
Hellenic Artillery School, Greece
May 2015

3rd INTERNATIONAL CONFERENCE ON CRYPTOGRAPHY, CYBER- SECURITY AND INFORMATION WARFARE
(http://sse.army.gr/sites/sse.army.gr/files/attachments/conference_announcement_in_english.pdf)
Lecture title: «*On the Mathematical Definition of Cyberspace*»
Hellenic Military Academy, Greece
May 2016

3rd INTERNATIONAL CONFERENCE ON CRYPTOGRAPHY, CYBER- SECURITY AND INFORMATION WARFARE
(http://sse.army.gr/sites/sse.army.gr/files/attachments/conference_announcement_in_english.pdf)
Lecture title: «*Mathematical Description of Cyber-Attacks and Proactive Defense*»
Hellenic Military Academy, Greece
May 2016

Employment in Research Center:Laboratoire d'Analyse Numérique et d'Optimisation, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France, 1984–2008

Participation in International Conferences: «THE SECONDTWO DAYS CONFERENCE INMATHEMATICALANALYSIS», National and Kapodistrian University of Athens, Greece,
February 1992

«INTERNATIONAL CONFERENCE ON NUMERICAL ALGORITHMS», Marrakech, Morocco,
October 2001

«INTERNATIONAL CONFERENCE ON COMPUTATIONAL METHODS IN SCIENCES AND ENGINEERING (ICCMSE) 2003», Chalkis, Greece,
September 2003

«INTERNATIONAL CONFERENCE ON APPROXIMATION AND ITERATIVE METHODS (AMI 2006)», Lille, France,
June 2006

«1st ATHENS COLLOQUIUM ON ALGORITHMS AND COMPLEXITY (ACAC 2006)», Athens, Greece,
August 2006

«INTERGATION OF MODELING AND SIMULATION», NATO Modeling and Simulation Group, Research and Technology Organization, Lecture Series MSG 067, Athens, Greece,
November 2007

«COMPUTATIONAL UNCERTAINTY IN MILITARY VEHICLE DESICN», NATO Applied Vehicle Technology Panel, Research and Technology Organization, AVT-147/RSY-022, Athens, Greece,
December 2007

«3èmes JOURNEES APPROXIMATION», Salle de Réunion, Bâtiment M2, Laboratoire Paul Painlevé UMR 8524, University of Sciences and Techniques of Lille Flandres-Artois, Lille, France,
May 2008

Supervision of Doctoral Theses

Alexopoulos Argyrios, Title of Doctoral Thesis: "*Cyberspace Modelling and Proactive Security"*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademy in collaboration with the Technical University of Crete in the object "APPLIED OPERATIONS RESEARCH & ANALYSIS", being prepared

Karamperas Nikolaos, Title of Doctoral Thesis: "*The transformation of supply chain management by integrating 3D printing in logistics. The military perspective"*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademyin collaboration with the Technical University of Cretein the object"APPLIED OPERATIONS RESEARCH & ANALYSIS", being prepared

Supervision of Postgraduate Theses

Karamperas Nikolaos, Title of Postgraduate Thesis: *“The Use of RFID Technology in Supply Chain Management and its Exploitation in Military Environment”*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademyin collaboration with the Technical University of Cretein the object"APPLIED OPERATIONS RESEARCH & ANALYSIS"), 2017

Passas Efstathios, Title of Postgraduate Thesis: “*Naval Combat Model as Decision Aiding Tool”*, Program of Postgraduate Studies being made the Hellenic Military Academy in collaboration with the Technical University of Cretein the object "APPLIED OPERATIONS RESEARCH & ANALYSIS"), 2018

Rizos Fotios, Title of Postgraduate Thesis: *“Cooperative Strategies of Missile Systems”*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademyin collaboration with the Technical University of Crete in the object"APPLIED OPERATIONS RESEARCH & ANALYSIS"), 2018

Argyriou Petros, Title of Postgraduate Thesis: *“Parametrization of Factors Affecting the Reliability of a Deliberate River Crossing Operation by a Modern Bridge Engineer Company and Model Construction”*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademyin collaboration with the Technical University of Crete in the object"APPLIED OPERATIONS RESEARCH & ANALYSIS"), 2018

Katti Helena, Title of Postgraduate Thesis: “*Prediction Models. Theoretical Overview and Applications”*, Program of Postgraduate Studies being made the Hellenic MilitaryAcademy in collaboration with the Technical University of Crete in the object"APPLIED OPERATIONS RESEARCH & ANALYSIS"), 2018

Supervision of Diploma Theses

Abelomos Athanassios, M.Sc., 2008, Title of Diploma Thesis: *Artificial war and multi-agent simulation of combat. The ISAAC-EINSTein program:Methodology*

Altimissis Haralambos, M.Sc., 2008, Title of Diploma Thesis: *Point and area targets in the no-defense case*

Bentis Nikolaos, M.Sc., 2008, Title of Diploma Thesis: *Prediction of combat outcomes. The ORSBM model: Mathematical background*

Broumidis Vassilios, M.Sc., 2008, Title of Diploma Thesis: *Offense and defense strategies for a group of identical targets*

Christou Ioannis, M.Sc., 2008, Title of Diploma Thesis: *Artificial war and multi-agent simulation of combat. The ISAAC-EINSTein program:Nonlinear dynamics, deterministic chaos and complex adaptive systems*

Dimou Ioannis, M.Sc., 2008, Title of Diploma Thesis: *Point and area targets in the no-defense case*

Grammatikopoulos Apostolos, M.Sc., 2008, Title of Diploma Thesis: *Offense and defense strategies for a group of identical targets*

Halkias Georgios, M.S., 2008, Title of Diploma Thesis: *Artificial war and multi-agent simulation of combat. The ISAAC-EINSTein program:Mathematical overview*

Hatzipanaghi Neokleous, M.Sc., 2008, Title of Diploma Thesis: *Defense of a target of unspecified structure*

Kakourakis Georgios, M.Sc., 2008, Title of Diploma Thesis: *Artificial war and multi-agent simulation of combat. The ISAAC-EINSTein program:Nonlinear Dynamics, deterministic chaos and complex adaptive systems*

Kakourakis Petros, M.Sc., 2008, Title of Diploma Thesis: *Artificial war and multi-agent simulation of combat. The ISAAC-EINSTein program:Nonlinear Dynamics, deterministic chaos and complex adaptive systems*

Kiriakou Savakis, M.Sc., 2008, Title of Diploma Thesis: *Defense of a target of unspecified structure*

Konstantinou Kiriakou, M.S., 2008, Title of Diploma Thesis: *Defense of a target of unspecified structure*

Kourtis Christos, M.Sc., 2008, Title of Diploma Thesis: *Point and area targets in the no-defense case*

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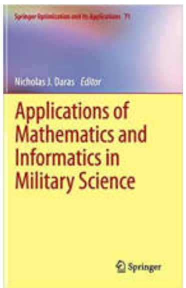

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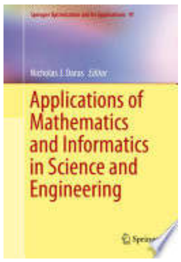

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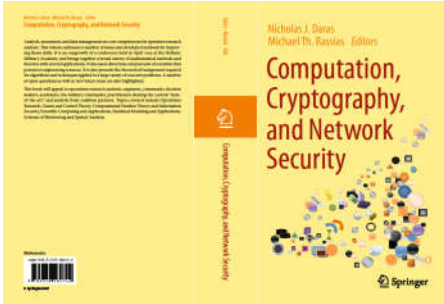

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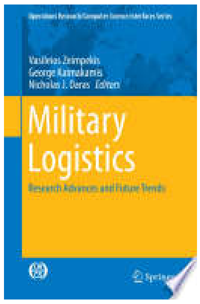

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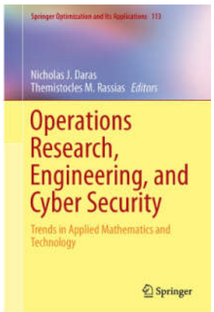

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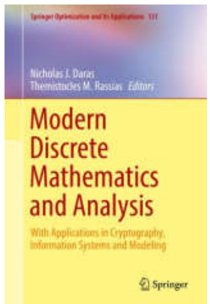

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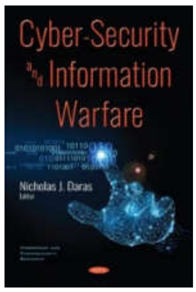

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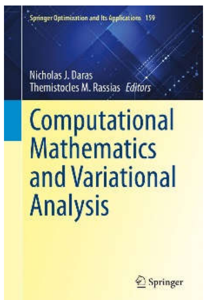

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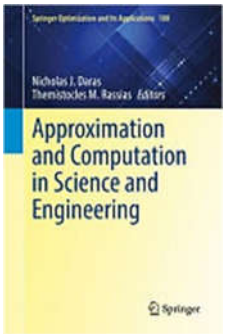

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2. N.J. Daras: **«Topologie et Calcul Différentiel»**, unpublished notes, 1990, pp.97
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4. N. J. Daras: **«Mathematical Analysis. Issue 1: Introduction»**, LectureNotes, Hellenic Military Academy, 2000, pp.327

Research Statement

Analysis of NicholasJ. Daras' Bibliography

Introduction

My scientific research project (and my total writing activity) consists in the following work.

- Fifty-seven (57) published, accepted, or completed papers (sole authorship in thirty-nine papers)
- Eighteen (18) books
- Twelve (12) edited books
- Five (5) editions of special issues in refereed journals
- Four (4) unpublished works
- Four (4) lecture notes.

This work is analyzed into twelve (12) general scientific directions:

- Numerical Analysis (*Rational approximation, rational interpolation, best rational approximation and interpolation theory, rational approximation to vectors of mutually irrational numbers, orthogonal polynomials in numerical reconstruction of signals, Markov-type inequalities in multivariate complex approximation,acceleration of computational schemes, interpolation methods for the numerical evaluation of finite Baire measures, multidimensional logarithmic residue formulasfor solving systems of non linear equations, complex extrapolated successive overrelaxation,numerical solving of differential equationsand integral equations, numerical evaluation of integrals and derivatives, numerical optimization*)
- Complex Analysis and Pure Mathematics (*Universal power series in one and several variables, summability transforms and analytic continuation in $\mathbb{C}^n$, biholomorhic maps in $\mathbb{C}^n$, integral representations in $\mathbb{C}^n$, contact geometry, topological and differential calculus, complex analysis,mathematical analysis, geometric analysis, projectiveanddescriptive geometry*)
- History of Mathematics (*History of continued fractions, connection to rational approximation theory, history of deterministic mathematical combat theories*)
- Stochastic Modeling and Numerical Simulation in Operations Research (*Stochastic and renewal combatmodels, reliability of military operations, missile allocation strategies and target coverage, numerical multi-agent simulation (in small to medium scale), strategic defense*)
- Quantum Computation & Cryptography (*Topological quantum computations, biholomorphic codes and cryptosystems*)
- Security (*Mathematical description of cyber attacks, modeling cyber-security, security and formation of network-centric operations, information security, stochastic analysis of cyber attacks, epidemiological diffusion and discrete branching models for malware propagation in computer networks*)
- Mathematical Modeling (*Modeling cyberspace, mathematical models in deterministic combat theory, ellipsoid targeting with overlap, mathematical models in portfolio analysis of defense, mathematical models in economy, mathematical models in fluid mechanics, mathematical models in military logistics*)
- Big Data (*Selective properties of big data*)

- **Data Management** (*Subjective preferences in data management*)
- **Prediction of Systemic Events** (*Prediction of peculiar systemic incidents, prediction of geopolitical events*)
- **Diophantine Equations** (Beal' s Conjecture, Generalized Fermat Equation)
- **Additive Number Theory** (Goldbach's Conjecture, Goldbach-Type Theorems).

In what follow, all the numbers that are enclosed in square brackets will refer to corresponding references of my publications, so as listed on pages 103–109, below.

Numerical Analysis

My research work on Numerical Analysis is directed towards thirteen(13) specialization areas.

Rational approximation

The first specialization focuses on ***rational approximation*** ([1],[2], [6], [8], [9], [12], [13], [15], [16], [17], [18], [19], [24], [28], [35] and [76]).

In my thesisentitled«Domaine de Convergence d'une Transformation de la Suite des Sommes Partielles d'une Fonction Holomorphe et Applications aux Approximants de Type Padé» ([1]), we consider Hermite polynomials of the Cauchy kernel

$$\prod_{j=1}^{n}\left(1-x_j z_j\right)^{-1}$$

to define and investigate rational approximants to multivariate Taylor series

$$f(z)=\sum_{\nu\in N^n} a_\nu z^\nu .$$

The free choice of the interpolation points may lead to a better approximation. To construct these approximants, it suffices to know only a few coefficients of this series. The algorithmic construction scheme and some indicative numerical examples are given in the paper «*The convergence of Padé-type approximants to holomorphic functions of several complex variables*» ([2]), where we also explore the local convergence behavior of a sequence of such approximants. The global convergence to holomorphic functions is studied extensively in the paper entitled «*Continuity of distributions and global convergence of Padé-type approximants in Runge domains»* ([6]).

The key idea for constructing such a rational approximant is to consider the $\mathbb{C}-$linear functional Λ_f associated with f and defined uniquely on the space $\mathcal{P}(\mathbb{C}^n)$ of all holomorphic polynomials by

$$\Lambda_f(x^\nu)=a_\nu,\ \nu=(\nu_1,\dots,\nu_n)\in N^n\ (|\nu|=\nu_1+\cdots+\nu_n).$$

If the series converges into an open polydisk $\Delta(0;r)$ of multiradius $r=(r_1,\dots,r_n)\in\mathbb{R}_+^n$ and center $0\in\mathbb{C}^n$, then by density (or by the Cauchy integral formula), Λ_f extends on the space of the functions which are homorphic in an open neighborhood of the closed polydisk $\Delta(0;r_1^{-1'},\dots,r_n^{-1})$ of center 0 and multiradius $(r_1^{-1},\dots,r_n^{-1})$, and there holds

$$f(z)=\Lambda_f\left(\prod_{j=1}^{n}\left(1-x_j z_j\right)^{-1}\right)\text{ for any } z=(z_1,\dots,z_n)\in\Delta(0;r).$$

If $p_m(x,z)=p_{m_1,\dots,m_n}(x_1,\dots,x_n,z_1,\dots,z_n)$ is the unique Hermite polynomial with degree at most $m=(m_1,\dots,m_n)$, which interpolates $\prod_{j=1}^{n}\left(1-x_j z_j\right)^{-1}$ at the $(m_1+1)+\cdots+(m_n+1)$ points

$$\left(\pi_{m_1,0}^{(1)},z_1\right),\left(\pi_{m_1,1}^{(1)},z_1\right),\dots,\left(\pi_{m_1,m_1}^{(1)},z_1\right),\dots,\left(\pi_{m_n,0}^{(n)},z_1\right),\left(\pi_{m_n,1}^{(n)},z_1\right),\dots,\left(\pi_{m_n,m_n}^{(n)},z_1\right)$$

the expression

$$\Lambda_f\left(p_m(x,z)\right)$$

is a function with numerator and denominator degrees at most

$$m = (m_1, \dots, m_n) \text{ and } m+1 = (m_1+1, \dots, m_n+1),$$

respectively. In fact, by setting

$$V_{m+1}(x_1, \dots, x_n) := \gamma \prod_{j=1}^{n} v_{m_j+1}(x_j),$$

with $\gamma \in \mathbb{C} \setminus \{0\}$ and $v_{m_j+1}(x_j) = \prod_{k_j=0}^{m_j} \left(x_j - \pi_{m_j,k_j}^{(j)}\right)$, it is easily seen that

$$\Lambda_f(p_m(x,z)) = \frac{\widehat{W}_m(z_1, \dots, z_n)}{\widehat{V}_{m+1}(z_1^{-1}, \dots, z_n^{-1})}$$

where we have used the notation

$$\widehat{W}_m(z_1, \dots, z_n) := z_1^{m_1} \dots z_n^{m_n} \Lambda_f \left(\frac{(-1)^{n+1} v_{m_1+1}(x_1) \dots v_{m_n+1}(x_n) + \cdots}{0} \right.$$

$$\frac{\dots + \sum_{i=1}^{n} v_{m_1+1}(z_1^{-1}) \dots v_{m_{i-1}+1}(z_{i-1}^{-1}) v_{m_i+1}(x_i) v_{m_{i+1}+1}(z_{i+1}^{-1}) \dots v_{m_n+1}(z_n^{-1})}{(x_1 - z_1^{-1}) \dots (x_n - z_n^{-1})}$$

$$\left. \frac{-v_{m_1+1}(z_1^{-1}) \dots v_{m_n+1}(z_n^{-1})}{0} \right),$$

and

$$\widehat{V}_{m+1}(z_1^{-1}, \dots, z_n^{-1}) := z_1^{m_1+1} \dots z_n^{m_n+1} V_{m+1}(z_1^{-1}, \dots, z_n^{-1}).$$

Thus, $\Lambda_f(p_m(x,z))$ is a rational function in $z = (z_1, \dots, z_n)$ of type$\left((m_1, \dots, m_n), (m_1 + 1, \dots\dots, m_n + 1)\right)$ which means that it has a numerator with degree in $z = (z_1, \dots, z_n)$ at most $(m_1, \dots, m_n)$ and a denominator with degree in $z = (z_1, \dots, z_n)$ at most $(m_1 + 1, \dots, m_n + 1)$. This rational function denoted by

$$(m/m+1)_f(z) = (m_1, \dots, m_n/m_1 + 1, \dots, m_n + 1)_f(z)$$

is characterized by the property:

$$f(z) - (m/m+1)_f(z) = \mathbf{O}\left(z_1^{m_1+1} \dots z_n^{m_n+1}\right)$$

and is known as a (multivariate) Padé-type approximant to the Taylor series $f(z)$, where as every polynomial $V_{m+1}(x_1, \dots, x_n)$ is called a *generating polynomial* of this approximation.

The main open question on Padé-type approximants is *the "best" choice of the interpolation points* $\pi_{m_1,0}^{(1)}, \pi_{m_1,1}^{(1)}, \dots, \pi_{m_1,m_1}^{(1)}, \dots, \pi_{m_n,0}^{(n)}, \pi_{m_n,1}^{(n)}, \dots, \pi_{m_n,m_n}^{(n)}$. Another problem, connected with the choice of the $\pi_{m_j,k}^{(j)}$'s is the problem of convergence of Padé-type approximants.

Despite any theoretical interest, the obtained formulas are of numerical utility only whenthe number n of the variables $z_1, \dots, z_n$is small enough. From a computational point of view, the same formulas become cumbersome and hardly manageable, if n is large.

One would adapt the simple proofs of the one variable to the several variables case, however three major obstacles present themselves. First, the local representation of a function holomorphic into a domain in $\mathbb{C}^n$ by its Taylor series may lead to complicated computations. Second, the polydisk $\Delta(0;r)$ does not qualify to be the general target domain because of the failure of the property to be the maximal domain of convergence of a multiple power series. Finally, there is no division process in $\mathcal{P}(\mathbb{C}^n)$, when $n > 1$. It is reasonable to suspect that the outlet lies with the consideration of another type of series representation for functions.

In this direction, in the paper «*Rational approximation to harmonic functions*» ([8]), we investigate rationalapproximation to the Fourier series expansion of a real-valued function u harmonic in the open unit disk D. By using the fundamental property of real harmonic functions to be the real parts of holomorphic functions in D, we define rational approximants $\Re(m/m+1)_u$ to u to be real parts of rational functions with degree at most m and denominator of degree at most $m+1$. The free choice of interpolation points may lead to a better approximation. The crucial *result we show is that thenumerator of such a rational function is determined by the condition that the Fourier series expansion of the restriction of such an approximant to any circle C_r of radius $r<1$ matches the Fourier series expansion of the restriction of the harmonic function to C_r as far as possible, that is up to the $\pm m$th Fourier term.*

Subsequently, in the paper «*Composed Padé-type approximation*» ([12]), we introduce composed (coordinate) rational approximations to complex-valued harmonic functions in D. The basic properties of such a coordinate approximation are being studied and we are showing that *any Padé-type approximant to a holomorphic function on D coincides with a composed rational approximant to this function*.

Then, the memoir «*Padé and Padé-type approximation for a 2π-periodic L^p –function*» ([9]) deals with Padé-type approximation to $2\pi-$periodic L^p functions on $[-\pi,\pi]$. The basic property we show is that *if the interpolation points lie in D, then the numerator of the Padé-type approximant of the function is completely determined by the condition thatits Fourier series expansion matches the Fourier series expansion of the function as far as possible, that is up to the $\pm m^{th}$ Fourier term.* Error expressions are given and several convergence results are proved with respect to the L^p- norm. Manynumerical examples are cited and*the relationship of Padé-type approximation withSchur's algorithm and thetheoryofSzegö's orthogonalpolynomials* is investigated. Finally, it is demonstrated *the possibility of unlimited improvement in a Padé-type approximation sequence pointwise convergence speed.*

In the paper *«Integral representations for Padé-type operators»* ([15]), we prove that *if, in particular, the periodic function is of class L^2 (or harmonic) and the interpolation points lie in D, then one can construct integral representations for theassociatedPadé-typeapproximants.* With thehelp oftheserepresentations,we obtain explicit forms for the Padé-type operators. These are integral operators associating a Padé -type approximant to each L^2 (or harmonic) function.Their usefulness lies intheir applicabilityto the convergence problem for a series of Padé-type approximants and the identification of theset of all periodic functions with the set of all the associated Padé-type approximants.

Having in mind all the results obtained in the case of one variable from the above work ([8], [9], [12] and [15]) we are in position to rethink the general case of several variables, discussed already in [1], [2] and [6].

To do so, in the paper *«Generalized Padé-type approximation and integral representation»* ([17]), we may replace the Cauchy kernel $\prod_{j=1}^{n}\left(1-x_j z_j\right)^{-1}$ by the Bergman kernel function $K_{\Omega}(z,x)$ into an open dounded subset Ω of $\mathbb{C}^n$ and, by using interpolating generalized polynomials for $K_{\Omega}(z,x)$, we define *generalized rational approximants* to any f in the space $\mathcal{O}L^2(\Omega)$ of all holomorphic functions on Ω which are of class L^2. The characteristic property of such an approximant is that *its Fourier series expansion with respect to an orthonormal basis for $\mathcal{O}L^2(\Omega)$ matches the Fourier series expansion of f as far as possible*. After studying the error formula and the convergence problem, we show that the generalized rationalapproximants have integral representations which give rise to the consideration of an integral operator –the so-called generalized Padé-typeoperator– which maps every $f\in\mathcal{O}L^2(\Omega)$ to a generalized Padé-type approximant to f. By continuity of this operator, we obtaine some convergence results about series of holomorphic functions of class L^2. Several numerical examples are given. Our study concludes with the extension of these ideas into every functional Hilbert space H and also with the definition and properties of the generalized Padé-type approximants to a linear operator of H into itself. As an application we prove a Painlevé-type theorem in $\mathbb{C}^n$.

Most of the avove results are also been collectedin the analyticaltreatise «Padé and Padé-Type Approximation to Fourier Series» ([35]).

- **Rational interpolation**

The second specialization of my work on Numerical Analysis is determined by the proof of a new *rational interpolation result*. More specifically, in the paper «*Padé and Padé-type approximation for a 2π-periodic L^P–function*» ([9]), westudy,among other,the Cauchy interpolation problem and determine a relationship between linearized rational interpolants and Padé-type approximants to holomorphic functions. Specifically, we prove the following.

Let f be a function homorphic in the open unit disk D. Let also $M = \left(\pi_{m,k}\right)_{m\geq 0, 0\leq k\leq m}$ is an infinite triangular interpolation matrix with $\pi_{m,k} \in D$. Suppose

$$\pi_{2m+1,m+1} = \pi_{2m+1,m+2} = \cdots = \pi_{2m+1,2m+1} = 0.$$

Set

$$u_{m+1}(x) := \prod_{k=0}^{m}\left(x - \pi_{2m+1,k}\right) \text{ and } \tilde{u}_{m+1}(z) := z^{m+1}u_{m+1}(z^{-1}).$$

(a) *If $r_{m,m+1} = (q_m/\tilde{u}_{m+1}) \in R_{m,m+1}(\mathbb{C})$ is a linearized rational interpolant to the homorphic function f at the $2m+2$ interpolation points of the set*

$$M_{2m+1} \coloneqq \left\{\pi_{2m+1,k} \colon k = 0,1,\dots,2m+1\right\},$$

then $r_{m,m+1}$ is a Padé-type approximant $(m/m+1)_f$ to f with generating polynomial $u_{m+1}(x)$.

(b) *Conversely, a Padé-type approximant $(m/m+1)_f$ to f with generating polynomial $u_{m+1}(x)$ is a linearized rational interpolant $r_{m,m+1} \in R_{m,m+1}(\mathbb{C})$ to f at the $2m+2$ interpolation points of the set $M_{2m+1} \coloneqq \left\{\pi_{2m+1,k} \colon k = 0,1,\dots,2m+1\right\}$, if there exists a $r < 1$ such that*

$$\int_{|s|=r}^{o} \frac{f(s)}{s^{2m+3}} \prod_{\substack{k=0\\(k\neq j)}}^{m} \left(1 - s\pi_{m,k}\right) ds = 0, \ (j = 0,1,2\dots,m).$$

- **Best rational approximation and interpolation theory**

The third specialization of my work on Numerical Analysis refers to the *"best" rational approximation problem and interpolation theory.*

In the paper *«On the best choice for the generating polynomial of a Padé-type approximation»* ([18]), we determine the best choice (pointwise, L^2 and uniform) of the interpolation points for a Padé-type approximation to a function holomorphic in an open planar disk D and give estimates for the uniform norm of the corresponding Padé-type approximation error. Specifically, we determine the "best" choice of points

$$\tilde{\pi}_0 = \tilde{\pi}_0(z), \tilde{\pi}_1 = \tilde{\pi}_1(z), \dots, \tilde{\pi}_m = \tilde{\pi}_m(z)$$

for the interpolation system $\pi_0, \pi_1, \dots, \pi_m$, in the sense that the corresponding Hermite polynomial $\tilde{p}_m(x,z)$ minimizes pointwise of z the absolute error

$$\left|f(z) - (m/m+1)_f(z)\right| \ (f \in \mathcal{O}\left(\Delta(0;r)\right)$$

into the ring $0 < |z| < r$, and, on the other hand, by showing that if m =even, the same as above choice constitutes also the "best" L^2 − choice, in the sense that it minimizes the L^2 −norm

$$\left\|f(z) - (m/m+1)_f(z)\right\|_2^{\delta.\varepsilon} \coloneqq \left(\int_{\delta\leq|z|<\varepsilon}^{0} \left|f(z) - (m/m+1)_f(z)\right|^2 dz\right)^{1/2}$$

of the error into an arbitrary half-open ring $\Delta(0;\delta,\varepsilon) := \{z \in C: \delta \le |z| < \varepsilon\}$, with $0 < \delta < \varepsilon < r$, over the subset of the Hermite polynomials $p_m(x,z)$, that is

$$\|f(z) - (m/m+1)_f(z)\|_2^{\delta.\varepsilon} = min_{\pi_0,\pi_1,\dots,\pi_m \in C}\|f(z) - (m/m+1)_f(z)\|_2^{\delta.\varepsilon}$$

$(f \in \mathcal{O}(\Delta(0;r))$. In both cases, the interpolation polynomial $\tilde{p}_m(x,z)$ has the form

$$\tilde{p}_m(x,z) = \sum_{\nu=0}^{m-1} z^\nu x^\nu + x^m \ (z \ne 0, x \in \mathbb{C}),$$

while the interpolation points $\tilde{\pi}_0 = \tilde{\pi}_0(z), \tilde{\pi}_1 = \tilde{\pi}_1(z), \dots, \tilde{\pi}_m = \tilde{\pi}_m(z)$ are the $(m+1)$ roots of the generating polynomial

$$\tilde{V}_{m+1}(x) = \tilde{V}_{m+1}^{(z)} = x^{m+1} + \frac{1}{z}(z^m - 1)x^m.$$

Furhter, if m =even, the Hermite polynomial $\tilde{p}_m(x,z)$ is the unique interpolation polynomial of degree at most m which minimizes the number

$$\left(\int_{\delta \le |z| < \varepsilon}^{0} \int_{|s|=\frac{1}{\rho}}^{0} \left|\frac{1}{1-sz} - p_m(x,z)\right|^2 dsdz\right)^{1/2}$$

and satisfies

$$\int_{\delta \le |z| < \varepsilon}^{0} \int_{|s|=\frac{1}{\rho}}^{0} \left(\frac{1}{1-sz} - p_m(x,z)\right) dsdz = 0 \ (0 < \delta < \varepsilon < \rho < r).$$

In spite, of these results, there is no analogous possibility to determine a “best” uniform choice for the interpolation system $\pi_0, \pi_1, \dots, \pi_m$, since a minimum for the uniform norm

$$\|f(z) - (m/m+1)_f(z)\|_\infty^{\delta.\varepsilon} = min_{\delta \le |z| < \varepsilon}|f(z) - (m/m+1)_f(z)| \ (f \in \mathcal{O}(\Delta(0;r))$$

of the error on a compact ring $\Delta(0;\acute{}\delta,\varepsilon)$ is obtained at the limit points

$$\pi_k = \infty \ (k = 0,1,\dots,m-i) \text{ and } \pi_\nu = 0 \ (\nu = m-i+1,\dots,m)$$

for any $i = 0,1,\dots,m+1$. In particular, among these choices, the only feasible optimal interpolation system is given by

$$\pi_0 = \pi_1 = \dots = \pi_m = 0.$$

- **Optimal rational approximation to vectors of mutually irrational numbers**

The fourth specialization of my work on Numerical Analysis deals with the problem of *optimal rational approximation to vectors of mutually irrational numbers*.

For a single irrational number$a$, it is well known that, whenever k is a natural number, there exists a rational approximant p/q with $q \le k$ and for which$|a - (p/q)| \le (1/kq)$. Especially, Dirichlet has shown that there are an *i n f i n i t e* number of irrational number of rational approximants p/q such that$|a - (p/q)| \le (1/q^2)$. In this direction, Hurwitz has shown that if $0 \le l \le \sqrt{5}$, then there are *i n f i n i t e l y* many rational approximants p/q satisfying$|a - (p/q)| \le (1/lq^2)$. This is an improvement of Dirichlet’s result. In particular, the extremity inequality$|a - (p/q)| \le (1/\sqrt{5}q^2)$ is best possible, since the golden ratio ϕ is irrational and if we replace $\sqrt{5}$ by any larger number $l(\sqrt{5})$ in the above expression then we will only be able to find *f i n i t e l y* many rational approximants p/q that satisfy the resulting inequality for $a = \phi$. However, Hurwitz also showed that if we omit the numberϕ, and numbers derived from it, then we can increase the number$\sqrt{5}$, in fact he showed we may replace it with$2\sqrt{2}$. Again this new bound is best possible in the new setting, but this time the number$\sqrt{2}$ is the problem. If we don’t allow$\sqrt{2}$, then we can increase the number on the right hand side of the inequality from$2\sqrt{2}$ to$\sqrt{221}/5$. Repeating this process we get an infinite sequence of numbers $\sqrt{5}, 2\sqrt{2}, \sqrt{221}/5, \dots$ which converge to3, called the Lagrange numbers.

The n[th] Lagrange number L_n is given by$L_n = \sqrt{9-(4/m_n^2)}$, where m_n is the nth Markov number, that is the nth smallest integer m such that the equation

$$m^2 + x^2 + y^2 = 3mxy$$

has a solution in positive integers x and y.

A central problem in number theory is how "best" approximate$a$ by a rational approximant(p/q). A natural way to define "best" rational approximation to$a$ is to take a rational number(p/q) that is closer to$a$ than any other rational approximant with a smaller denominator$q$. With this definition of "best" rational approximation, the answer to the above question was solved long ago by the method of continued fractions: the convergent of the continued fraction expansion of$a$ give the "best" rational approximants to$a$. Specifically, the only way a fraction can approximate$a$ better than a convergent is if the fraction has a bigger denominator than the convergent. So, in spite of its simple formulation, the fraction "best" approximating$a$ depends strongly on the denominator of the convergent and thus, from a numerical point of view, one might expect that the "best" rational approximants lack most of their practical usefulness appeal.

There is another independent reason advocating for this expectation. It is perhaps surprising that the innocent generalization of this problem to the simultaneous rational approximants to several mutually irrational numbers is considered a difficult, essentially unsolved problem in number theory. The precise generalization of rational approximants to a single irrational number is to define a sequence

$$\left\{\sigma^{(k)} = \left(\sigma_1^{(k)}, \sigma_2^{(k)}, \dots, \sigma_n^{(k)}\right) = \left(\frac{p_1^{(k)}}{r^{(k)}}, \frac{p_2^{(k)}}{r^{(k)}} \dots, \frac{p_n^{(k)}}{r^{(k)}}\right) \in Q^n : k = 1,2, \dots\right\}$$

of ordered sets of n rational numbers $\sigma_j^{(k)} = \left(p_j^{(k)}/r^{(k)}\right)$ $(j = 1,2, \dots, n)$ each set with common denominator $r^{(k)} \in \mathbb{Z} \setminus \{0\}$, which converges to a given $n-$vector of mutually irrational numbers $a = (a_1, a_2, \dots, a_n) \in \mathbb{A}^n \equiv (\mathbb{R} \setminus \mathbb{Q})^n$. Obviously, in order to define "best" rational $n-$vector approximation in the above sense, we have to consider a metric measuring the distance of such a rational $n-$vector approximant $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n) = \left((p_1/r), (p_2/r), \dots, (p_n/r)\right)$ to $a = (a_1, a_2, \dots, a_n)$. There are two common natural metrics in use: the metric which measures "weak convergence" and the metric which measures "strong convergence". According to Khinchin, a rational $n-$vector approximant can only be called "best" relative to a particular metric[2]. But, in spite of its theoretical interest, the "best" relativeness deprives of a standard criterion describing numerical approximation efficacy. In this direction, Kim and Ostlund gave rational rational approximants to pairs of mutually irrational numbers and showed that binary Facey tree organization of the rationals extends naturally to a binary organization of pairs of rationals with a common denominator[3]. The rational approximants generated by their algorithm are in fact the best rational approximants generated by the criterion of weak convergence. However, the Kim-Ostlund algorithm does not always give a "best" rational approximant for arbitrary simultaneous irrationals. Following Kim and Ostlund, "it appears that we must give up either the best rational approximants or give up scaling of approximations. Giving up some of the best rational approximants seems less harmful to analysis if it is scaling and renormalization which will be important to dynamical systems theory".

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2 See A.Y. Khinchin: Continued Fractions, University of Chicago, Chicago 1964.

3 See S.Kim and S. Ostlund: Simultaneous rational approximations in the study of dynamical systems, Physical Review A, Volume 34, Issue 4, 1981, Pages 3426-3434

In the paper *«Optimal rational approximation number sets: Application to nonlinear dynamics in particle accelerators»* ([41]), we show how rational approximation theory can be cleared of its dependence on “best” approximants and reconnected to original ideas of n u m e r i c a l a p p r o x i m a t i o n. More precisely, we propose a numerical method for constructing simultaneous rational approximations. To do so, we investigate multivariate rational approximation numbers whose decimal expansion string of digits coincides with the decimal expansion digital string mutually irrational numbers as far as possible.

The suggested method proceeds in two main steps. First, we consider the n irrational coordinates of the vector $a = (a_1, a_2, \dots, a_n) \in \mathbb{A}^n$ expressed in the decimal system:

$$a_j = a_j^{(0)}.a_j^{(1)}a_j^{(2)} \dots a_j^{(m)} \dots = a_j^{(0)} + \frac{a_j^{(1)}}{10} + \frac{a_j^{(2)}}{10^2} + \dots + \frac{a_j^{(m)}}{10^m} + \cdots$$

($a_j^{(0)} \in \mathbb{Z}$, $a_j^{(\nu)} \in \mathbb{N}$, $0 \le a_j^{(\nu)} \le 9$ for $\nu = 1,2,\dots$ and $j = 1,2,\dots,n$). The associated power series with integral coefficients $f_j(z) = a_j^{(0)} + a_j^{(1)}z + a_j^{(2)}z^2 + \dots + a_j^{(m)}z^m + \cdots$ converges compactly (:uniformly on every compact set) into the open unit disk D of the complex plan $\mathbb{C}$. Next, we approximate the series $f_j(z)$ by using Padé and Padé-type approximants to $f_j(z)$ in $\Delta(0;1)$. Recall that Padé approximants are rational functions whose expansion in ascending powers of the variable z coincides with the Taylor power series expansion of the analytic function $f_j(z)$ into D as far as possible, that is up to the sum of the degrees of the numerator and denominator. The numerator and denominator of a Padé approximant are completely determined by this condition and no freedom is left. Thus, Padé approximation to $f_j(z)$ is the rational function analogue to the Taylor polynomial approximation to $f_j(z)$. Padé-type approximants to $f_j(z)$ are rational functions with an arbitrary denominator, whose numerator is determined by the condition that the expansion of the Padé-type approximant matches the Taylor series expansion of $f_j(z)$ into the disk D as far as possible, that is up to the degree of the numerator. The great advantage of Padé-type over Padé approximants lies in the free choice of the poles which may lead to a better approximation. Especially, we constructed rational approximants to the series $f_j(z)$ at the point$z = (1/10)$.

The resulting rational approximation numbers have decimal expansions which coincide with the decimal expansion $a_j^{(0)}.a_j^{(1)}a_j^{(2)} \dots a_j^{(m)} \dots$ of the irrational number a_j as far as possible. Repetition of this coordinate procedure with only formal changes to substitute a_j by every one of the other irrational coordinates $a_1, a_2, \dots, a_{j-1}, a_{j+1}, \dots, a_n$ of $a \in A^n$ gives sequences

$$(OPTANUS)_a = \left\{ (OPTANUS/k)_a = \left(pt_1 = \frac{\hat{W}_k^{(1)}}{\hat{V}_{k+1}}, \dots, pt_n = \frac{\hat{W}_k^{(n)}}{\hat{V}_{k+1}} \right) \in \mathbb{Q}^n : k = 1,2,\dots \right\}$$

of ordered sets of n rational numbers with decimal expansions matching simultaneously the decimal expansions of the corresponding n irrational numbers $a = (a_1, a_2, \dots, a_n) \in A^n$ as far as possible.

The convergence of such a sequence $(OPTANUS)_a$ to a, as well as the best choice of the common denominator $\hat{V}_{k+1}$, are studied in [41]. Notice that there is no restriction on the choice of the denominators: in contrast to the initially described situation of ordered sets with common denominators, the only criterion imposed here is the long coincidence of the decimal coordinate expansions.

Orthogonal polynomials and numerical reconstruction of signals

The next specialization of my work on Numerical Analysis is specified by the *numerical reconstruction of signals and orthogonal polynomials theory.*

Given a signal, say a sound or an image, Fourier analysis easily calculates the frequencies and the amplitudes of those frequencies which make up the signal. However, Fourier methods are not always a good tool to recapture the signal, particularly if it is highly non-smooth: too much Fourier information is needed to reconstruct the signal locally. Especially, there is no analytical way to reconstruct with exactitude a signal if only a few of its initial Fourier coefficients are known.

Such extension problems in Taylor series context have a long history dating from 1907. Indeed, the Schur problem, or Carathéodory-Fejér problem, were to find conditions for the existence of an analytic function bounded by one in the unit planar disk whose initial Taylor coefficients are given numbers$a_0, a_1, \dots, a_\mu$. Schur showed that such a function exists if and only if the lower triangular matrix

$$\left(a_{\mu,k} = a_{\mu-k}\right)_{k\le\mu} = \begin{pmatrix} a_0 & \cdots & 0 & 0 \\ a_1 & \ddots & \dots & 0 \\ \vdots & \ddots & a_0 & \vdots \\ a_\mu & \cdots & a_1 & a_0 \end{pmatrix}$$

is bounded by one as an operator on complex Euclidean space, and he determined how all solutions can be found. The method was adapted to Pick-Nevanlinna interpolation by Nevanlinna. Ever since, several different applications and analogue developments are considered. For instance, given a partial covariance sequence of lengthμ, the problem of finding all positive rational extensions of degree at most μ is a fundamental open problem with important applications in signal processing and speech processing and in stochastic realization theory and system identification.

In *«Numerical reconstruction of signals and orthogonal polynomials»* ([24]) and «*Orthonormality in interpolation schemes for reconstructing signals*» ([28]), we consider *a numerical version of the above Carathéodory-Schur interpolation problem in the trigonometric context*. In particular, *we investigate a numerical method for constructing efficient approximants to any continuous-time periodic signal by using only a few of its initial Fourier coefficients*. These approximants are real parts of rational functions with numerators determined by the condition that the Fourier series expansion of the approximants matches the Fourier series of the signal as far as possible.

The detailed definition of these approximants to a continuous-time periodic L^1 −signal and their principal properties are presented. Their convergence depends strongly on the orthonormality of the chosen generating polynomial system $\{V_{m+1}(e^{it}) : m = 0,1,\dots\}$ into$L^2[-\pi,\pi]$. *The form of each $V_{m+1}(x)$is characterized by recurrence relations dues to the connection between Schur and Szegö theories*. Further, we study the general assumptions under which, for every sequence of functions converging to a periodic continuous signal there is an approximation sequence converging pointwise to that signal faster than the first sequence.

Finally, several numerical examples are given.

Markov-type inequalities in multivariate complex approximation

In *«Markov's property and generalized Padé-type approximants»* ([16]) and the paper *«Generalized Padé-type approximants to continuous functions»* ([19]), we discussthe possibility of expandingthe definitionand properties oftheseapproximantsin the caseof the spaceof continuous functionson a compactset thatsatisfiesa conditionMarkov. More precisely, we define generalized rational approximants to continuous functions on a compact subset E of $\mathbb{R}^n$ satisfying Markov's inequality and we show that thw Fourier series expansion of a generalized Padé-type approximant to a $u \in C^\infty(E)$ matches the Fourier series expansion of u as far as possible. After studying the errors, we give integral representations and an answer to the convergence problem of a generalized rational approximation sequence.

Finally, in [35], we give a brief and comprehensive overview of all therefinements, extensions and generalizations of the Markov-type inequalities and we summarize the basic applications of various forms of these inequalities in multivariate approximation theory. In [35], we provided a brief overviewof several refinements and applications of the Markov-type inequalities in various contexts. To this end, let $\mathcal{P}_d(\mathbb{K})$ *be the collection of all polynomials of degree at most* d *with coefficients in the field* $\mathbb{K} = \mathbb{R}, \mathbb{C}$. An inequality of the Markov-type is an inequality of the form $\|P^{(\nu)}(x)\| \leq C\|P(x)\|$ for every $P \in \mathcal{P}_d(\mathbb{K})$. Here $P^{(\nu)}(x)$ denotes the ν th derivative. The best possible constant C depends on d, ν and the norm $\|\cdot\|$, and the determination and estimation of C has been the subject of numerous investigations since Andrei Markov's paper [50][4] (:$\nu = 1$ and $\|\cdot\|$ being the L^∞ −norm on a bounded interval)and the paper[5] by his brother Vladimir Markov: ($\nu \geq 2$ and the same L^∞ −norm). A.A. Markov's original paper dates back to 1889 and it is not readily accessible[6]. For a modern exposition on this and other related topics we refer to the standard bibliography[7]. The first purpose of [35] is to give a brief overview of all the refinements, extensions and generalizations of the Markov-type inequalities. The second purpose is to summarize some basic applications of various forms of these inequalities in the theory of approximation.

Acceleration of computational schemes

The seventh specialization of my work on Numerical Analysis is specified by the *study of the acceleration of a given computational scheme*.

In the paper «*Padé and Padé-type approximation for a 2π-periodic* L^p *–function*» ([9]) (see also the treatise «Padé and Padé-Type Approximation to Fourier Series» [76]), we study the assumptions under which, for any given sequence of functions converging to a real-valued continuous 2π −periodic function on$[-\pi, \pi]$, there is always a Padé-type approximation sequence converging point-wise to that function faster than the first sequence. This property, due to the free choice of the interpolation points$\pi_{m,k}$, permits us to construct better and better approximations to continuous functions. More specifically, we obtain the following three answers on the convergence acceleration problem.

Theorem 2 ([9]). *Suppose there are a constant* $K > 0$*and an open neighbourhood* U*of the unit circle into which the generating polynomials* $V_{m+1}(x)$*of a Padé-type approximation satisfy* $K \leq |V_{m+1}(z)|$, *for any* $z \in U$*and any* m *sufficiently large. Further, assume that the family* $\{V_{m+1}(e^{it}): m = 0,1, \ldots\}$*is orthonormal in* $L^2[-\pi, \pi]$. *If*

$$\overline{lim}_{m\to\infty}\left\{sup_{|z|\leq 1}\left|\frac{V_{m+1}(x)V_{m+2}(e^{-it})-V_{m+1}(e^{-it})V_{m+2}(x)}{1-xe^{it}}\right|\right\}^{1/m} = R(t), \ (t \in [-\pi, \pi]),$$

then, for any real-valued continuous 2π −*periodic function*f *on* $[-\pi, \pi]$, *the corresponding sequence* $\{Re(m/m+1)_f(t): m = 0,1, \ldots\}$ *of Padé-type approximants to* f *converges to* $f(t)$ *faster than any strictly monotone converging sequence* $\{y_m: m = 0,1, \ldots\}$ *satisfying*

$$lim_{m\to\infty}|\Delta y_m|^{1/m} > R(t).$$

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[4] A. A. Markov: *On a problem of D. I. Mendeleev*, Zap. Im. Akad. Nauk. 62(1889), 1-24

[5] V. Markov: "*Über die Funktionen, die in einem gegebenen Intervall möglichst wenig von Null abweichen*", Math. Ann. 77 (1916), 213-258

[6] A. A. Markov: *On a problem of D. I. Mendeleev*, Zap. Im. Akad. Nauk. 62(1889), 1-24.

[7] R. P. Boas: Inequalities for the derivatives of polynomials, *Math. Mag.* 42(1969), 165-174.

Theorem 3 ([9]). *Let*

$$V_{m+1}(x) = \gamma \prod_{k=0}^{m}\left(x - \pi_{m,k}\right) = \sum_{k=0}^{m+1} b_k^{(m)} x^k \ (m = 0,1, \dots)$$

be the generating polynomials of a Padé-type approximation such that

$$\sum_{k=0}^{m+1} \left|b_k^{(m)}\right|^2 = \frac{1}{2\pi}(m = 0,1, \dots),$$
$$\sum_{k=0}^{m+1} b_k^{(m)} \acute{b}_k^{(n)} = 0(m < n) \text{ and}$$
$$\overline{\lim}_{m\to\infty}\left[\sum_{k=0}^{m+2}\sum_{\substack{k=0\\(\nu\neq k)}}^{m+1}\left|b_k^{(m+1)} b_\nu^{(m)}\right|\right]^{1/m} = R.$$

If there are two constants $\sigma < \infty$ *and* $c < 1$ *fulfilling*

$$\sum_{k=0}^{m+1}\left|b_k^{(m)}\right| < \sigma(m = 0,1, \dots) \text{ and } \left|\pi_{m,k}\right| < c \ (m \geq 0 \text{ and } 0 \leq k \leq m),$$

then, for any real-valued continuous 2π *–periodic function* f *on* $[-\pi,\pi]$, *the corresponding sequence* $\{Re(m/m+1)_f(t): m = 0,1, \dots\}$ *of Padé-type approximants to* f *converges to* $f(t)$ *everywhere in* $[-\pi,\pi]$, *faster than any strictly monotone converging sequence* $\{y_m: m = 0,1, \dots\}$ *satisfying*

$$\lim_{m\to\infty}|\Delta y_m|^{1/m} > R.$$

Theorem 4 ([9]). *Assume that the generating polynomials* $V_{m+1}(x)$ *of a Padé-type approximation satisfy*

$$\lim_{m\to\infty}\frac{V_{m+1}(x)}{V_{m+1}(z^{-1})} = 0$$

compactly in an open set $\omega \subset \mathbb{C}^2$ *containing* $(\overline{D} \times D) \cup (\mathbb{C} \times \{0\})$. *If*

$$\overline{\lim}_{m\to\infty}\left\{sup_{|z|\leq 1}\left|\frac{V_{m+1}(x)V_{m+2}(e^{-it}) - V_{m+1}(e^{-it})V_{m+2}(x)}{1 - xe^{it}}\right|\right\}^{1/m} = R(t), \ (t \in [-\pi,\pi])$$

then, for any real-valued continuous 2π *–periodic function* f *on* $[-\pi,\pi]$, *the corresponding sequence* $\{Re(m/m+1)_f(t): m = 0,1, \dots\}$ *of Padé-type approximants to* f *converges to* $f(t)$ *faster than any strictly monotone converging sequence* $\{y_m: m = 0,1, \dots\}$ *satisfying*

$$\lim_{m\to\infty}|\Delta y_m|^{1/m} > R(t).$$

- **Interpolation methods for the numerical evaluation of finite Baire measures**

  In the paper *«Interpolation methods for the evaluation of a 2π-periodic finite Baire measure»* ([13]), we define and study rational approximation to 2π –periodic finite Baire measures on $[-\pi,\pi]$. As in [9], the basic property we show is that *if the interpolation points lie in* D, *then the numerator of the rational approximant of the measure is completely determined by the condition thatits Fourier series expansion matches the Fourier series expansion of the measure as far as possible, that is up to the* $\pm m$*th Fourier term*. But, in contrast to [9], totally different error expressions are obtained and new convergence results are given with respect to the weak star topology of measures.

- **Multidimensional logarithmic residue formulas for solving systems of non linear equations**

  My ninth specialization in Numerical Analysis is directed towards *numerical algorithms, notably multidimensional logarithmic residue formulas,for solving systems of non linear* *equations.*Any method for numerical solving systems of nonlinear equations makes use of initial points which are arbitrarily close to unknown solutions of these systems. However, in most cases, there is no possibility of pre-locating solutions, so a suitable choice for initial points becomes totally risked.

In the papers *«Detection of initial points of methods solving homogeneous polynomial systems»* ([56]) and *«A Direct numerical algorithm for Solving Systems of Non Linear Equations using multidimensional logarithmic residue formulas»* ([57]), we describe and study an efficient algorithm for approximating solutions of algebraic systems. The algorithm uses multidimensional logarithmic residue integral formulas. We interpret this algorithm as a fast and reliable new method for locating initial points of well known numerical methods. For this purpose, we combine the algorithm with well known iterative schemes and investigate convergence rate and complexity properties of the resulting combined methods.

Let us briefly outline the constitutive idea of the intended methods. Suppose the system of $n$algebraic equations

$$\begin{cases} f_1(z_1, z_2, \dots, z_n) = 0 \\ \vdots \\ f_n(z_1, z_2, \dots, z_n) = 0 \end{cases} \tag{1}$$

in n unknowns $z_1, z_2, \dots, z_n$has at most finitely many discrete roots (: common discrete zeros) in an *arbitrarily* chosen open subset Ω of $\mathbb{C}^n$, say N. Denote these roots by

$$\xi^{(1)} = \left(\xi_1^{(1)}, \xi_2^{(1)}, \dots, \xi_n^{(1)}\right), \cdots, \xi^{(N)} = \left(\xi_1^{(N)}, \xi_2^{(N)}, \dots, \xi_n^{(N)}\right).$$

Then multidimensional analogues of the classical logarithmic residue of Cauchy can be applied to compute sums of the form

$$\sum_{v=1}^{N} \varphi\left(\xi^{(v)}\right) = \varphi\left(\xi^{(1)}\right) + \cdots + \varphi\left(\xi^{(N)}\right)$$

where

- φ is *any* function in the space $A(\Omega) \cap C(\bar{\Omega})$ of all functions which are holomorphic in Ω and can be extended continuously on $\bar{\Omega}$, and
- each term $\varphi\left(\xi^{(v)}\right)$ is taken as many times as the multiplicity of $\xi^{(v)}$.

The large amount of arbitrariness inφ's choice can be used for obtaining interesting results. If for instance $\varphi = \varphi(z_1, z_2, \dots, z_n) \equiv 1$, then *the corresponding multidimensional integral over the boundary* $\partial\Omega$ *of* Ω

$$\int_{\partial\Omega} \omega\left(F(z), F\acute{(}z)\right) \tag{2}$$

equals the number of roots (*counting multiplicity*) *in* Ω, i.e. to N. Here $\omega\left(F(z), \overline{F(z)}\right)$ is the differential form

$$\omega\left(F(z), \overline{F(z)}\right) := \frac{(n-1)!}{(2\pi i)^n} \frac{1}{\|F\|^{2n}} \sum_{j=1}^{n} (-1)^{j-1} \bar{f}_j d\bar{F}_{[j]} \wedge dF,$$

with

$$d\bar{F}_{[j]} \wedge dF := d\bar{f}_1 \wedge \dots \wedge d\bar{f}_{j-1} \wedge d\bar{f}_{j+1} \wedge \dots \wedge d\bar{f}_n \wedge df_1 \wedge \dots \wedge df_n.$$

Similarly, if we set

$$\varphi = \varphi(z_1, z_2, \dots, z_n) \equiv z_j^p \,(p = 1,2, \dots, N \wedge j = 1,2, \cdots, n),$$

then computation of the corresponding multidimensional integrals over $\partial\Omega$

$$S_p^{(j)} := \int_{\partial\Omega} z_j^p \, \omega\left(F(z), \overline{F(z)}\right) (p = 1,2, \dots, N)$$

enables us to replace (1) by a family of n independent systems each of which consisting in N (algebraic) equations of the form

$$\begin{cases} \sum_{\nu=1}^{N} \xi_1^{(\nu)} = S_1^{(1)} \\ \sum_{\nu=1}^{N} \left(\xi_1^{(\nu)}\right)^2 = S_2^{(1)} \\ \vdots \\ \sum_{\nu=1}^{N} \left(\xi_1^{(\nu)}\right)^N = S_N^{(1)} \end{cases} \quad \begin{cases} \sum_{\nu=1}^{N} \xi_2^{(\nu)} = S_1^{(2)} \\ \sum_{\nu=1}^{N} \left(\xi_2^{(\nu)}\right)^2 = S_2^{(2)} \\ \vdots \\ \sum_{\nu=1}^{N} \left(\xi_2^{(\nu)}\right)^N = S_N^{(2)} \end{cases}, \cdots\cdots, \begin{cases} \sum_{\nu=1}^{N} \xi_n^{(\nu)} = S_1^{(n)} \\ \sum_{\nu=1}^{N} \left(\xi_n^{(\nu)}\right)^2 = S_2^{(n)} \\ \vdots \\ \sum_{\nu=1}^{N} \left(\xi_n^{(\nu)}\right)^N = S_N^{(n)} \end{cases} \tag{3}$$

,

With this formulation, we can now proceed in two steps:

- In a first step, the problem is reduced to the simpler one of finding the roots

$$\xi_1^{(1)}, \xi_1^{(2)}, \cdots, \xi_1^{(N)}, \xi_2^{(1)}, \xi_2^{(2)}, \cdots, \xi_2^{(N)}, \cdots, \xi_n^{(1)}, \xi_n^{(2)}, \cdots, \xi_n^{(N)}$$

of n polynomials

$$\pi^{(1)}(\xi), \pi^{(2)}(\xi), \cdots, \pi^{(n)}(\xi).$$

In fact, assuming *thatthemultidimensional integrals* $S_p^{(1)}, \cdots, S_p^{(n)} (p = 2, \cdots, N)$ *in* (3) *are computed with sufficient accuracy*,the coefficients

$$\alpha_1^{(1)}, \alpha_2^{(1)}, \ldots, \alpha_{N-1}^{(1)}, \alpha_N^{(1)}, \alpha_1^{(2)}, \alpha_2^{(2)}, \ldots, \alpha_{N-1}^{(2)}, \alpha_N^{(2)}, \cdots, \alpha_1^{(n)}, \alpha_2^{(n)}, \ldots, \alpha_{N-1}^{(n)}, \alpha_N^{(n)}$$

of the polynomials

$$\begin{gathered} \pi^{(1)}(\xi) \equiv \xi^N + \alpha_1^{(1)} \xi^{N-1} + \cdots + \alpha_{N-1}^{(1)} \xi + \alpha_N^{(1)}, \\ \pi^{(2)}(\xi) = \xi^N + \alpha_1^{(2)} \xi^{N-1} + \cdots + \alpha_{N-1}^{(2)} \xi + \alpha_N^{(2)} \\ \vdots \\ \pi^{(n)}(\xi) = \xi^N + \alpha_1^{(n)} \xi^{N-1} + \cdots + \alpha_{N-1}^{(n)} \xi + \alpha_N^{(n)} \end{gathered}$$

are expressed in terms of the power sums

$$\begin{gathered} S_1^{(1)}, S_2^{(1)}, \cdots, S_N^{(1)}, \\ S_1^{(2)}, S_2^{(2)}, \cdots, S_N^{(2)} \\ \vdots \\ S_1^{(n)}, S_2^{(n)}, \cdots, S_N^{(n)} \end{gathered}$$

via *Newton's recursion formula* $p\alpha_p^{(j)} = -S_p^{(j)} - S_{p-1}^{(j)} \alpha_1^{(j)} - \cdots - S_1^{(j)} \alpha_{p-1}^{j}$ (for $p = 1,2, \ldots, N$ and $j = 1,2, \ldots, n$):

$$\begin{cases} \alpha_1^{(1)} = -S_1^{(1)} \\ 2\alpha_2^{(1)} = -S_2^{(1)} - S_1^{(1)} \alpha_1^{(1)} \\ 3\alpha_3^{(1)} = -S_3^{(1)} - S_2^{(1)} \alpha_1^{(1)} - S_1^{(1)} \alpha_2^{(1)} \\ \vdots \\ (p-1)\alpha_{p-1}^{(1)} = -S_{p-1}^{(1)} - S_{p-2}^{(1)} \alpha_1^{(1)} - \cdots - S_1^{(1)} \alpha_{p-2}^{(1)} \\ p\alpha_p^{(1)} = -S_p^{(1)} - S_{p-1}^{(1)} \alpha_1^{(1)} - \cdots - S_1^{(1)} \alpha_{p-1}^{(1)} \end{cases}$$

(4)

$$\begin{cases} \alpha_1^{(2)} = -S_1^{(2)} \\ 2\alpha_2^{(2)} = -S_2^{(2)} - S_1^{(2)} \alpha_1^{(2)} \\ 3\alpha_3^{(2)} = -S_3^{(2)} - S_2^{(2)} \alpha_1^{(2)} - S_1^{(2)} \alpha_2^{(2)} \\ \vdots \\ (p-1)\alpha_{p-1}^{(2)} = -S_{p-1}^{(2)} - S_{p-2}^{(2)} \alpha_1^{(2)} - \cdots - S_1^{(2)} \alpha_{p-2}^{(2)} \\ p\alpha_p^{(2)} = -S_p^{(2)} - S_{p-1}^{(2)} \alpha_1^{(2)} - \cdots - S_1^{(2)} \alpha_{p-1}^{(2)} \end{cases}$$

$$\ddots \quad \ddots$$

$$\begin{cases} \alpha_1^{(n)} = -S_1^{(n)} \\ 2\alpha_2^{(n)} = -S_2^{(n)} - S_1^{(n)}\alpha_1^{(n)} \\ 3\alpha_3^{(n)} = -S_3^{(n)} - S_2^{(n)}\alpha_1^{(n)} - S_1^{(n)}\alpha_2^{(n)} \\ \vdots \\ (p-1)\alpha_{p-1}^{(n)} = -S_{p-1}^{(n)} - S_{p-2}^{(n)}\alpha_1^{(n)} - \cdots - S_1^{(n)}\alpha_{p-2}^{(n)} \\ p\alpha_p^{(n)} = -S_p^{(n)} - S_{p-1}^{(n)}\alpha_1^{(n)} - \cdots - S_1^{(n)}\alpha_{p-1}^{(n)} \end{cases}$$

- In a second step, the numbers

$$\xi_1^{(1)}, \xi_1^{(2)}, \cdots, \xi_1^{(N)}, \xi_2^{(1)}, \xi_2^{(2)}, \cdots, \xi_2^{(N)}, \cdots, \xi_n^{(1)}, \xi_n^{(2)}, \cdots, \xi_n^{(N)}$$

being all successfully determined, the problem is reduced to the simpler one of checking the simultaneous validity of the given nonlinear equations

$$\begin{cases} f_1(\xi_1, \xi_2, \dots, \xi_n) = 0 \\ \vdots \\ f_n(\xi_1, \xi_2, \dots, \xi_n) = 0 \end{cases} \tag{5}$$

for any n −ple $(\xi_1, \xi_2, \dots, \xi_n)$ such that

  - $\xi_1 \in \left\{\xi_1^{(1)}, \xi_1^{(2)}, \cdots, \xi_1^{(N)}\right\}$,
  - $\xi_2 \in \left\{\xi_2^{(1)}, \xi_2^{(2)}, \cdots, \xi_2^{(N)}\right\}$,

  ....

  - $\xi_n \in \left\{\xi_n^{(1)}, \xi_n^{(2)}, \cdots, \xi_n^{(N)}\right\}$.

The n −ple $(\xi_1, \xi_2, \dots, \xi_n)$ satisfying (5) will be the N roots of (1) into the open subset Ω of $\mathbb{C}^n$. Notice that, in finding the N roots $\xi_j^{(1)}, \xi_j^{(2)}, \cdots, \xi_j^{(N)}$ of the polynomial $\pi^{(j)}(\xi) \equiv \xi^N + \alpha_1^{(j)}\xi^{N-1} + \cdots + \alpha_{N-1}^{(j)}\xi + \alpha_N^{(j)}$ $(j = 1,2, \dots, n)$, we have eliminated all the unknowns except one, *with no superfluous roots added and no roots missing*.

Of course, the main difficulty in this approach is the computation of the multidimensional integrals $S_p^{(j)}$ $(p = 1,2, \dots, N, j = 1,2, \dots, n)$. The ingredient arbitrariness in the applied integral formulas (that is not present in the one dimensional case) enables us to choose suitable expressions for the logarithmic residue so that the integrals $S_p^{(j)}$ can be computed exactly.

Complex extrapolated successive overrelaxation

The tenth specialization of my work on Numerical Analysis relates to the *iterativemethods for solvingsystems of linear equations*. More specifically, in the paper *«On complex extrapolated successive overrelaxation(ESOR): some theoretical results»* ([11]), we discuss the complex extrapolated successive overrelaxation (ESOR) method for the numerical solution of large sparse linear systems of complex algebraic equations. Some subsets of convergence for this method are obtained through an application of conformal mapping techniques.

Numerical solving of differential equations and integral equations

The eleventh specialization of my work on Numerical Analysis is directed towards the study of *new algorithms for the numerical solving of ordinary differential equations and integral equations*.

Let

$$\sum_{k=0}^{m} p_k(t) \frac{d^k f}{dt^k}(t) = g(t) \; (p_m(0) \neq 0)$$

be any nonhomogeneous linear differential equation with real analytic coefficients $p_k(t)$ and $g(t)$. If the coefficients are constants:

$$p_k(z) = a_k = constant \text{ and } g(z) = d = constant,$$

then the above equation becomes

$$\sum_{k=0}^{m} a_k \frac{d^k f}{dt^k}(t) = d \ (a_m(0) \neq 0).$$

In the paper *«Rational approximate solutions to linear differential equations»* ([22]), we give the following direct numerical algorithm to solve these equations.

Assume that

$$p_k(z) = \sum_{\nu=0}^{\infty} a_\nu^{(k)} z^\nu \text{ and } g(z) = \sum_{\nu=0}^{\infty} d_\nu z^\nu.$$

Suppose only the M+1 first in order Taylor coefficients

$$d_0, d_1, \dots, d_M$$

of the analytic function $g(t)$ are known.

For each $j = 0,1,\dots,m$, **choose** real numbers $c_0^{(j)}$, $c_1^{(j)}, \dots,$ $c_m^{(j)}$, so that the vectors

$$\begin{pmatrix} c_0^{(0)} - c_0^{(1)} \\ c_1^{(0)} - c_1^{(1)} \\ \vdots \\ c_{m-1}^{(0)} - c_{m-1}^{(1)} \end{pmatrix}, \begin{pmatrix} c_0^{(0)} - c_0^{(2)} \\ c_1^{(0)} - c_1^{(2)} \\ \vdots \\ c_{m-1}^{(0)} - c_{m-1}^{(2)} \end{pmatrix}, \dots, \begin{pmatrix} c_0^{(0)} - c_0^{(m)} \\ c_1^{(0)} - c_1^{(m)} \\ \vdots \\ c_{m-1}^{(0)} - c_{m-1}^{(m)} \end{pmatrix}$$

are linearly independent in $\mathbb{R}^m$.

Let t be any point in $(-r,r) \setminus \{0\}$.

I. Method for differential equations with analytic coefficients

1. **For each** $j = 0,1,\dots,m$ **and** $N = 0,1,\dots,M$ **put**

$$c_{m+N}^{(j)} = \frac{1}{m!\, a_0^{(m)} \binom{-m-1}{N}} \Big[(-1)^N d_N - \sum_{i=0}^{N-1} (-1)^{N+i} \sum_{k=i}^{m+i} (k+i)!\, a_{N-i}^{(k-i)} \binom{-k+i-1}{i} c_k^{(j)} - (-1)^N \sum_{k=N}^{m+N-1} (k-N)!\, a_0^{(k-N)} \binom{-k+N-1}{N} c_k^{(j)}\Big];$$

2. **Define** $n := m + M$;
3. **For** $j = 0,1,\dots,m$ **and** $\nu=0,1,\dots,n$, **put**

$$T_j(x^\nu) = c_\nu^{(j)}$$

and **choose**

$$\pi_{n,\nu}^{(j)}(t) \in C \text{ with } \left[\pi_{n,\nu}^{(j)}(t)\right]^{-1} \neq t;$$

4. **For** $j = 0,1,\dots,m$, **define**
   - $V_{j,n+1}^{(t)}(x) := \gamma_j \prod_{\nu=0}^{n} \left(x - \pi_{n,\nu}^{(j)}(t)\right)$, $(\gamma_j \in \mathbb{C} \setminus \{0\})$
   - $\hat{W}_{j,n}^{(t)}(t) := t^n T_j\left([x - t^{-1}]^{-1} / \left[V_{j,n+1}^{(t)}(x) - V_{j,n+1}^{(t)}(t^{-1})\right]\right)$
   - $\hat{V}_{j,n+1}^{(t)}(t) := t^{n+1} V_{j,n+1}^{(t)}(t^{-1})$

- $[(PTSA)_n]_j(t) \coloneqq \left(\hat{W}_{j,n}^{(t)}(t) / \hat{V}_{j,n+1}^{(t)}(t)\right)$
- $\left[(PTSA)_n^{(j)}\right](t) \coloneqq [(PTSA)_n]_0(t) - [(PTSA)_n]_j(t)$;

5. **A rational approximate solution to the homogeneous linear differential equation**

$$\sum_{k=0}^{m} p_k(t) \frac{d^k f}{dt^k}(t) = 0 \ (p_m(0) \neq 0)$$

is given by

$$[(PTSA)_n](t) = \sigma_1\left[(PTSA)_n^{(1)}\right](t) + \cdots + \sigma_m\left[(PTSA)_n^{(m)}\right](t)$$

$(\sigma_1, \dots, \sigma_m \in R)$

6. **A rational approximate solution to the nonhomogeneous linear differential equation**

$$\sum_{k=0}^{m} p_k(t) \frac{d^k f}{dt^k}(t) = g(t) \ (p_m(0) \neq 0)$$

is given by

$$[(PTSA)_n](t) + [(PTSA)_n]_0(t)$$

End.

II. Method for differential equations with constant coefficients

1. **For each** $j = 0,1, \dots, m$ **and** $N = 0,1, \dots, M$ **put**

$$c_{m+N}^{(j)} = \frac{1}{m!a_0^{(m)}\binom{-m-1}{N}}\left[(-1)^N d_N - \sum_{k=0}^{m-1} k!\, a_k c_{k+N}^{(j)} \binom{-k-1}{N}\right]$$

2. **Define** $n := m = M$;
3. **For** $j = 0,1, \dots, m$ **and** ν=0,1,…,n, **put**

$$T_j(x^\nu) = c_\nu^{(j)}$$

and **choose**

$$\pi_{n,\nu}^{(j)}(t) \in C \text{ with } \left[\pi_{n,\nu}^{(j)}(t)\right]^{-1} \neq t;$$

4. **For** $j = 0,1, \dots, m$, **define**
   - $V_{j,n+1}^{(t)}(x) \coloneqq \gamma_j \prod_{\nu=0}^{n}\left(x - \pi_{n,\nu}^{(j)}(t)\right)$, $(\gamma_j \in \mathbb{C} \setminus \{0\})$
   - $\hat{W}_{j,n}^{(t)}(t) \coloneqq t^n T_j\left([x - t^{-1}]^{-1} / \left[V_{j,n+1}^{(t)}(x) - V_{j,n+1}^{(t)}(t^{-1})\right]\right)$
   - $\hat{V}_{j,n+1}^{(t)}(t) \coloneqq t^{n+1} V_{j,n+1}^{(t)}(t^{-1})$
   - $[(PTSA)_n]_j(t) \coloneqq \left(\hat{W}_{j,n}^{(t)}(t) / \hat{V}_{j,n+1}^{(t)}(t)\right)$
   - $\left[(PTSA)_n^{(j)}\right](t) \coloneqq [(PTSA)_n]_0(t) - [(PTSA)_n]_j(t)$
5. **A rational approximate solution to the homogeneous linear differential equation**

$$\sum_{k=0}^{m} a_k \frac{d^k f}{dt^k}(t) = 0 \ (a_m(0) \neq 0)$$

is given by

$$[(PTSA)_n](t) = \sigma_1\left[(PTSA)_n^{(1)}\right](t) + \cdots + \sigma_m\left[(PTSA)_n^{(m)}\right](t)$$

$(\sigma_1, \dots, \sigma_m \in \mathbb{R})$

6. **A rational approximate solution to the nonhomogeneous linear differential equation**

$$\sum_{k=0}^{m} a_k \frac{d^k f}{dt^k}(t) = g(t)\ (a_m(0) \neq 0)$$

is given by

$$[(PTSA)_n](t) + [(PTSA)_n]_0(t)$$

End.

As for the numerical solution of integral equations, we recall that one of the most effective methods for the numerical solution of integral equations is the method of replacing the integral equation by a system of linear algebraic equations using a quadrature formula. Suppose we are given the equation

$$f(t) - \lambda \int_{-\pi}^{\pi} h(t,s) f(s) \phi(s) ds = g(t). \qquad (6)$$

If we replace the integral, using a numerical interpolation formula based on points $t_1, t_2, \dots, t_N$ and require (6) to be satisfied only at these points, then we obtain the linear system

$$f(t_j) - \lambda \sum_{k=1}^{N} A_k h(t_j, t_k) f(t_k) \phi(t_k) = g(t_j),\ j = 1,2,\dots,N$$

any solution of which determines an approximate value for the required solution at the points $t_1, t_2, \dots, t_N$.

It often happens that the integral in (6) is considered with respect to some finite measure μ on $[-\pi, \pi]$:

$$f(t) - \lambda \int_{-\pi}^{\pi} h(t,s) f(s) d\mu(s) = g(t). \qquad (7)$$

In the paper *«Interpolation methods for the evaluation of a 2π-periodic finite Baire measure»* ([13]) we are concerned with constructing a general approximation method for a large class of measures measure μ on $[-\pi, \pi]$, in such a way that one can approximate to an integral equation (7) by replacing it by an equation of the type (6).

To see this, let us consider any finite real Baire measure μ on $[-\pi, \pi]$. For definiteness, we assume that μ is 2π −periodic, i.e., if μ has a point mass at $-\pi$ or π, these masses must be the same:

$$\mu(\{-\pi\}) = \mu(\{\pi\}).$$

Then, μ can be regarded as a measure on the unit circle C, obtained by identifying $-\pi$ and π, and the Poisson integral $u(re^{it}) = u(z)$ of μ is a function, real-valued and harmonic in z. From the solution of the Dirichlet problem in the unit disk D, it follows that, when $r \to 1$, the measures

$$d\mu_r(t) = u_r(t) dt \qquad (u_r(t) = u(re^{it}))$$

converges to $d\mu(t)$ in the weak-star topology on measures.

By using interpolation methods, we seek for an effective approximation to $u_r(t)$. A natural approach to its solution is affroaded by the ideas of rational approximation theory. We can therefore approximate $d\mu_r(t)$ by a sequence

$$\{Re(m/m+1)_u(re^{it}) dt: m = 0,1,2,\dots\}$$

where $Re(m/m+1)_u(z)$ is a Padé-type approximant to the harmonic function $u(z)$. When $r \to 1$, the measures $Re(m/m+1)_u(re^{it})dt$ converge to the finite real measure

$$Re(m/m+1)_\mu(t) dt \coloneqq Re(m/m+1)_u(e^{it}) dt$$

in the weak-star topology on measures. The boundedness in L^1 −norm of the family

$$\{Re(m/m+1)_u(re^{it}) dt: 0 \le r < 1\}$$

guarantees that the Fourier series expansion of the limit measure $\Re(m/m+1)_\mu(t)dt$ matches the Fourier series expansion of μ up to the $\pm m^{th}$ Fourier-Stieljes term. The measure

$$Re(m/m+1)_\mu(t) dt$$

is called *a Padé-type approximant to* $d\mu(t)$; the integral equation

$$f(t) - \lambda \int_{-\pi}^{\pi} h(t,s) f(s) Re(m/m+1)_{\mu}(s) ds = g(t)$$

is called a *a Padé-type approximate equation to* (7).

To judge the effectiveness of these methods, and the extent to which they can be justified, the methods need to be investigated theorically. In [13], we discuss the convergence of a Padé-type approximation sequence to a finite Baire measure with respect to general orthonormality properties. In particular, we confine ourselves to the fruitful case of a finite 2π −periodic Baire measure on $[-\pi, \pi]$ and give various sufficient convergence conditions.

Numerical evaluation formulas for definite integrals and derivatives

The twelfth specialization of my work on Numerical Analysis is the investigation of *new approximate formulas for the numerical evaluation of definite integrals and derivatives*.

Let f *be any real-valued continuous function in the interval* $-\pi \leq t \leq \pi$, *with* $f(-\pi) = f(\pi)$. *Suppose* f' *exists and is piecewise continuous on the interval* $-\pi < t < \pi$.

In the paper «*Padé and Padé-type approximation for a 2π-periodic* L^p*–function*» ([9]) (see also the treatise «Padé and Padé-Type Approximation to Fourier Series» [76]), we prove the following statement.

Theorem 1 ([9]). *Let* $M = \left(\pi_{m,k}\right)_{m \geq 0, 0 \leq k \leq m}$ *is an infinite triangular interpolation matrix with* $\pi_{m,k} \in D$. *For each* $m = 0,1,2,\dots$, *define the polynomial*

$$V_{m+1}(x) = \prod_{k=0}^{m} \left(x - \pi_{m,k}\right).$$

Let also

$$\sum_{-\infty}^{\infty} c_{\nu} e^{i\nu t}$$

be the Fourier series representation of f.

[1] *Define the* $\mathbb{C}$ −*linear fuctional*

$$T_f : \mathcal{P}(\mathbb{C}) \to \mathbb{C} : T_f(x^{\nu}) := c_{\nu}.$$

The derivative

$$f'(t)$$

of f *can be approximated by the expression*

$$-2Re\left[\frac{ie^{it}}{V_{m+1}(e^{-it})}\left\{T_f\left(\frac{xV'_{m+1}(x)}{e^{-it}-x}\right) + T_f\left(xe^{-it}\frac{V_{m+1}(e^{-it}) - V_{m+1}(x)}{\left(e^{-it}-x\right)^2}\right)\right\}\right] \quad (m = 0,1,2,\dots)$$

The error of such an approximation is given by

$$\frac{1}{\pi} \lim_{r \to 1} Re\left[i \int_{-\pi}^{\pi} \frac{f(s)}{re^{it} - e^{is}} \frac{V'_{m+1}(e^{-is})}{V_{m+1}(r^{-1}e^{-it})}\right]$$

where the limit is uniform on $[-\pi, \pi]$.

[2] *Define the* C −*linear functional*

$$T_{\int f} : \mathcal{P}(\mathbb{C}) \to \mathbb{C} : T_f(x^{\nu}) := \begin{cases} \frac{c_{\nu}}{i\nu}, if \nu \geq 1 \\ 0, if \nu = 0 \end{cases}.$$

The definite integral

$$\int_{t_0}^{t} f(s) ds$$

of f *can be approximated by the expression*

$$c_0(t-t_0)+2Re\left[\frac{e^{it}}{V_{m+1}(e^{-it})}T_{\int f}\left(\frac{V_{m+1}(e^{-it})-V_{m+1}(x)}{e^{-it}-x}\right)\right.$$
$$\left.-\frac{e^{it_0}}{V_{m+1}(e^{-it_0})}T_{\int f}\left(\frac{V_{m+1}(e^{-it_0})-V_{m+1}(x)}{e^{-it_0}-x}\right)\right]$$
$$(m=0,1,2,\ldots)$$

The error of such an approximation is given by

$$\lim_{r\to 1}Re\left[\frac{1}{V_{m+1}(r^{-1}e^{-it})}T_{\int f}\left(\frac{V_{m+1}(x)}{xre^{-it}-1}\right)-\frac{1}{V_{m+1}(r^{-1}e^{-it_0})}T_{\int f}\left(\frac{V_{m+1}(x)}{xre^{-it_0}-1}\right)\right]$$

where the limit is uniform on $[-\pi,\pi]$.

- **Writings**

  Besides the above specifications,my work on the generalfield of Numerical Analysis includes teaching monographs.

  The three volumes of the monograph entitled «Elements of Numerical Analysis» ([86], [87], [88]) quote basic algorithmic tools and develop sufficient theoretical background for a complete advanced undergraduate course of Numerical Analysis.

  The lecture notes «Mathematical Programming:Theory and Algorithms.Issue 1:Linear Programming» ([77]), «Mathematical Programming:Theory and Algorithms.Issue 2: the ellipsoid and Karmarkar' Algorithms»([78]) and «Numerical Optimization» ([79])provide provide a sufficient introductory material for a graduate course in mathematical programming algorithms with particular emphasis on numerical optimization methods.

  Finally, in the monographentitled «Approximation Complexe Quantitative» ([98]) we discuss thegeneral problemof constructingnumericalapproximations tocomplex functions.

Complex Analysis

My research work on Complex Analysis is directed towards five (5) main specializations: *universal power series in one and several variables, summability transforms and analytic continuation in* $\mathbb{C}^n$*, biholomorhic mappings in* $\mathbb{C}^n$ and *integral representations in* $\mathbb{C}^n$.

- **Universal power series in $\mathbb{C}$ and $\mathbb{C}^n$**

  The first specialization of my work on Complex Analysis focuses on *universal power series* ([25], [29], [40] and [50]).

  In his thesis, under Charles Hermite, Henri Padé arranged these approximants in a double array now known as the Padé table of the formal power series$f=\sum_{\nu=0}^{\infty}a_\nu z^\nu=(a_0,a_1,\ldots)$:

  $$\left([p/q]_f\right)_{p,q=0,1,2,\ldots}.$$

  One of the fascinating features of Padé approximants is the complexity of their asymptotic behaviour. The convergence problem for Padé approximants can be stated as follows. Given a power series $f=\sum_{\nu=0}^{\infty}a_\nu z^\nu=(a_0,a_1,\ldots)$ examine the convergence of subsequences $\left([p_n/q_n]_f : n=0,1,2,\ldots\right)$extracted from the Padé table as $n\to+\infty$. If f is the Taylor development of an entire function, then it is known that generically there exists a sequence$(p_n,q_n)_{n=0,1,2,\ldots}$,$q_n>0$, so that $lim_{n\to\infty}[p_n/q_n]_f=f$.

In the paper «*Universal Padé approximation*» ([25]), we investigate all the possible limits of sequences

$$\left([p_n/q_n]_f : n = 0,1,2,\ldots\right)$$

on compact subsets K of $\mathbb{C} \setminus \{0\}$ or compact sets disjoint from the domain of definition of the holomorphic function f. We show that, generically, *all functions holomorphic in a neighborhood of* K *are such limits*. Thus, *we have formal power series* (*or holomorphic functions on a domain* Ω) *with universal Padé approximants*. The particular case $q = 0$ is the well known case of universal Taylor series where the approximation is realized by the partial sums. However, now we impose several conditions on the approximating integers (p_n, q_n) and q_n may be different from0; in particular, we can have universal approximations with $p_n = q_n$, or $lim_{n\to\infty} p_n = lim_{n\to\infty} q_n = +\infty$ or $lim_{n\to\infty}(p_n - q_n) = +\infty$ and others. We obtain the universal approximation by Padé approximants on compact sets K with connected complement K^c.

However, *the fact that Padé approximants may also have poles allows us to do approximation on compact sets* K *of arbitrary connectivity* and these results are generic on spaces of holomorphic functions defined on arbitrary planar domains and not just on simply connected domains. This is not possible in the case of universal Taylor series, where the approximation is realized by polynomials (the partial sums). As methods of proofs we use Baire's Category Theorem combined with Runge's or Mergelyan's Theorems.

Further, in the paper «*Padé approximation of Seleznev type*» ([29]), we use the chordal metric in order to approximate all meromorphic functions on $\mathbb{C} \setminus \{0\}$ by Padé approximants of formal power series. This is a generic universality of Seleznev type which implies Menchoff type almost everywhere approximation with respect to any σ-finite Borel measure on $\mathbb{C} \setminus \{0\}$. Let us explain the obtained results. First, we make the notation needed. Let $\zeta \in \mathbb{C}$ be fixed and $f = \sum_{\nu=0}^{\infty} a_\nu (z-\zeta)^\nu$ be a formal power series ($a_\nu = a_\nu(f,\zeta) \in \mathbb{C}$). Often this series comes from the Taylor with center of expansion ζ of a holomorphic function on an open set containingζ. For $p, q \in \{0,1,2,\ldots\}$, we define the Padé approximant $[p/q]_{f,\zeta}(z)$ to be a rational function $\phi(z) = A(z)/B(z)$, where A, B are polynomials satisfying $degA \le p$, $degB \le q$, $B(z_0) \ne 0$, whose Taylor expansion $\phi(z) = \sum_{\nu=0}^{\infty} b_\nu (z-\zeta)^\nu$ is such that $b_\nu = a_\nu$ for all $\nu = 0,1,\ldots,p+q$. For$q = 0$, the Padé approximant $[p/0]_{f,\zeta}(z)$ exists trivially and it is uniquely equal to $[p/0]_{f,\zeta}(z) = \sum_{k=0}^{p} a_k (z-\zeta)^k$. For$q \ge 1$, it is not always true that $[p/q]_{f,\zeta}(z)$ exists, or that it is unique if it does exist. A necessary and sufficient condition for existence and uniqueness is that the following $q \times q$ Hankel at a_{p-q+1} is different from zero:

$$det\underbrace{\begin{pmatrix} a_{p-q+1} & a_{p-q+2} & a_{p-q+3} & \cdots & a_p \\ a_{p-q+2} & a_{p-q+3} & a_{p-q+4} & \cdots & a_{p+1} \\ a_{p-q+3} & a_{p-q+4} & a_{p-q+5} & \cdots & a_{p+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_p & a_{p+1} & a_{p+2} & \cdots & a_{p+q-1} \end{pmatrix}}_{q} \ne 0$$

where $a_i = 0$ for $i < 0$. Whenever the latter holds, we write

$$f \in \mathfrak{D}_{p,q}(\zeta).$$

In this case, the Padé approximant

$$[p/q]_{f,\zeta}(z) = \frac{A(f,\zeta)(z)}{B(f,\zeta)(z)}$$

is given explicitly by the following Jacobi formula:

$$\mathcal{A}(f,\zeta)(z) = \\ det\begin{pmatrix} (z-\zeta)^q S_{p-q}(f,\zeta)(z) & (z-\zeta)^{q-1} S_{p-q+1}(f,\zeta)(z) & \dots & S_p(f,\zeta)(z) \\ a_{p-q+1} & a_{p-q+2} & \dots & a_{p+1} \\ \vdots & \vdots & \dots & \vdots \\ a_p & a_{p+1} & & a_{p+q} \end{pmatrix},$$

$$\mathcal{B}(f,\zeta)(z) = det\begin{pmatrix} z^q & z^{q-1} & \dots & 1 \\ a_{p-q+1} & a_{p-q+2} & \dots & a_{p+1} \\ \vdots & \vdots & \dots & \vdots \\ a_p & a_{p+1} & & a_{p+q} \end{pmatrix}$$

with

$$S_k(f,\zeta)(z) = \begin{cases} \sum_{\nu=0}^{k} a_\nu (z-\zeta)^\nu, ifk \geq 0 \\ 0, ifk < 0. \end{cases}$$

If $\mathcal{A}(f,\zeta)(z)$ and $\mathcal{B}(f,\zeta)(z)$ are given by the previous Jacobi formula and they do not have a common zero on a set K, we write

$$f \in \mathcal{E}_{p,q,\zeta}(K).$$

If K is compact, then $f \in \mathcal{E}_{p,q,\zeta}(K)$ is equivalent to the existence of a $\delta > 0$ so that

$$|\mathcal{A}(f,\zeta)(z)|^2 + |\mathcal{B}(f,\zeta)(z)|^2 > \delta \text{ for all } z \in K.$$

It is natural for us to consider the chordal metric χwhich allows the existence of poles on the domain where the approximations hold. The chordal metric$\chi$is defined on $\mathbb{C}\cup\{\infty\}$:

$$\chi(a,b) = \begin{cases} \frac{|a-b|}{\sqrt{1+|a|^2}\sqrt{1+|b|^2}}, ifa,b \in \mathbb{C} \\ \frac{1}{\sqrt{1+|a|^2}}, ifa \in \mathbb{C} \\ 0, ifa = \infty, b = \infty \end{cases}.$$

Let also Ω be an open subset of $\mathbb{C}$. We define the set

$$\mathfrak{M}(\Omega) = \left\{f\colon \Omega \to \mathbb{C}\cup\{\infty\}\colon on\ each\ component\ V\ of\ \Omega\ either\ {}^{f}/_{V} \equiv \infty\ or\ {}^{f}/_{V}\ is\ meromorphic\right\}.$$

The space of all formal power series is denoted by $\mathbb{C}^{\aleph_0}$. The main results in [29] are given below.

Theorem 5 ([29]). *Let* $\mathfrak{F}$ *be a subset of* $\mathbb{N}\times\mathbb{N}$ *containing a sequence* $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2,\ldots$, *with* $\tilde{p}_n \to +\infty$ *and* $\tilde{q}_n \to +\infty$. *Then there exists a formal power series* $f = \sum_{\nu=0}^{\infty} a_\nu z^\nu$, $(a_\nu) \in \mathbb{C}^{\aleph_0}$, *with the following property: For any compact* $K \subset \mathbb{C}\setminus\{0\}$ *and any* $h(z) \in \mathfrak{M}(\mathbb{C}\setminus\{0\})$ *there exists a sequence* $(p_n, q_n) \in F$, $n = 1,2,\ldots$, *such that*

- $f \in \mathfrak{D}_{p_n,q_n}(0)$ *and* $f \in \mathcal{E}_{p_n,q_n,0}(K)$ *for every* $n \in N$ *and*
- $sup_{z\in K}\chi\left([p_n/q_n]_{f,0}(z), h(z)\right) \to 0$, *as* $n \to +\infty$.

The set of all f *with this property is dense and* G_δ *in* $C^{\aleph_0}$ *endowed with the Cartesian topology.*

We now endow the space $\mathbb{C}^{\aleph_0}$ with a metric ϱ_d giving a topology different from the Cartesian one and we get similar results to the above one in the space $(\mathbb{C}^{\aleph_0}, \varrho_d)$. For any two formal power series $f = \sum_{\nu=0}^{\infty} a_\nu z^\nu$ and $g = \sum_{\nu=0}^{\infty} b_\nu z^\nu$, we define

$$\varrho_d(f,g) = 2^{-\kappa}$$

where

$$\kappa = \begin{cases} \min\{\nu = 0,1,2, \dots : a_\nu \neq b_\nu\}, if\ f \neq g \\ \infty. if\ \ f = g \end{cases}.$$

One can see that

$$(\mathbb{C}^{\aleph_0}, \varrho_d)$$

is a complete metric space and therefore Baire's Category theorem is in our disposal.

In analogy to Theorem 5, we prove the following result.

Theorem 6 ([29]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$ and $\tilde{q}_n \to +\infty$. Then there exists a formal power series $f = \sum_{\nu=0}^{\infty} a_\nu z^\nu$, $(a_\nu) \in \mathbb{C}^{\aleph_0}$, with the following property: For every compact $K \subset \mathbb{C} \setminus \{0\}$ and for every $h(z) \in \mathfrak{M}(\mathbb{C} \setminus \{0\})$ there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that*

- *$f \in D_{p_n,q_n}(0)$ and $f \in E_{p_n,q_n,0}(K)$ for every $n \in N$ and*
- *$sup_{z\in K} \chi\left([p_n/q_n]_{f,0}(z), h(z)\right) \to 0$, as $n \to +\infty$.*

The set of all f with this property is dense and G_δ in$(\mathbb{C}^{\aleph_0}, \varrho_d)$.

Theorem 7 ([29]). *Let f_0 be a fixed formal power series which satisfies theorem 5 or 6. Let also μ be a $\sigma-$finite Borel measue on $\mathbb{C} \setminus \{0\}$. Then, for every $\mu-$measurable function $h: \mathbb{C} \setminus \{0\} \to \mathbb{C}$ there exists a sequence $(p_n, q_n) \in F$, $n = 1,2, \dots$, such that*

- *$f_0 \in D_{p_n,q_n}(0)$ and*
- *$[p_n/q_n]_{f,0}(z) \xrightarrow[n\to+\infty]{} h(z)$ $\quad \mu-$almost everywhere.*

Theorem 8 ([29]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$ and $\tilde{q}_n \to +\infty$. Then there exists a formal power series $f = \sum_{\nu=0}^{\infty} a_\nu z^\nu$, $(a_\nu) \in \mathbb{C}^{\aleph_0}$, with the following property: For every compact $K \subset \mathbb{C} \setminus \{0\}$ with K^c connected and every $\varphi(z) \in A(K) := \{g: K \to \mathbb{C}: g \text{ continuous on } K \text{holomorphic on} K^0\}$ there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that*

- *$f \in \mathfrak{D}_{p_n,q_n}(0)$ and $f \in \mathcal{E}_{p_n,q_n,0}(K)$ for every $n \in \mathbb{N}$ and*
- *$sup_{z\in K}\left|[p_n/q_n]_{f,0}(z) - \varphi(z)\right| \to 0$, as $n \to +\infty$.*

The set of all f with this property is dense and G_δ in $\mathbb{C}^{\aleph_0}$ endowed with the Cartesian topology.

Theorem 9 ([29]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$ and $\tilde{q}_n \to +\infty$. Then there exists a formal power series $f = \sum_{\nu=0}^{\infty} a_\nu z^\nu$, $(a_\nu) \in \mathbb{C}^{\aleph_0}$, with the following property: For every compact $K \subset \mathbb{C} \setminus \{0\}$ with K^c connected and every function $\varphi(z) \in A(K)$ there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that*

- *$f \in D_{p_n,q_n}(0)$ for every $n \in N$ and*
- *$sup_{z\in K}\left|[p_n/q_n]_{f,0}(z) - \varphi(z)\right| \to 0$, as $n \to +\infty$.*

The set of all f with this property is dense and G_δ in $\mathbb{C}^{\aleph_0}$ endowed with the metric ϱ_d.

Theorem 10 ([29]). *Let* f_0 *be a fixed formal power series which satisfies theorem 8 or 9. Let also* μ *be a* $\sigma-$*finite Borel measue on* $\mathbb{C}\setminus\{0\}$. *Then, for every* $\mu-$*measurable function*

$$h: \mathbb{C}\setminus\{0\}\to\mathbb{C}$$

there exists a sequence $(p_n, q_n)\in\mathfrak{F}$, $n=1,2,\ldots$, *such that*

- $f_0\in D_{p_n,q_n}(0)$ *and*
- $[p_n/q_n]_{f,0}(z)\xrightarrow[n\to+\infty]{} h(z)$ $\mu-$*almost everywhere.*

In the paper «*Universal Padé approximants on simply connected domains*» ([50]), we present a generic extension of the above results in a simply connected domain Ω. The universal approximation is required only on compact sets K which lie outside Ω and have connected complement. If the sets K are additionally disjoint from the boundary of Ω, then the universal functions can be smooth on the boundary. Specifically, we prove the following results.

Theorem 11 ([50]). *Let* $\mathfrak{F}$ *be a subset of* $\mathbb{N}\times\mathbb{N}$ *containing a sequence* $(\tilde{p}_n,\tilde{q}_n)\in\mathfrak{F}$, $n=1,2,\ldots$, *with* $\tilde{p}_n\to+\infty$.*Let* $\Omega\subset\mathbb{C}$ *be a simply connected open set and* L *and* L' *two compact subsets of* Ω *(with connected complements). Let* $K\subset\mathbb{C}\setminus\Omega$ *be another compact set with connected complement. Then there exists a holomorphic function*$f\in\mathcal{O}(\Omega)$, *such that, for every polynomial* h *there exists a sequence* $(p_n,q_n)\in\mathfrak{F}$, $n=1,2,\ldots$, *such that the following apply:*

- $f\in\mathfrak{D}_{p_n,q_n}(\zeta)$ *and* $f\in\mathcal{E}_{p_n,q_n,\zeta}(L'\cup K)$ *for every* $n\in\mathbb{N}$, $\zeta\in L$,
- $sup_{\zeta\in L, z\in K}\left|[p_n/q_n]_{f,\zeta}(z)-h(z)\right|\to 0$, *as* $n\to+\infty$ *and*
- $sup_{\zeta\in L, z\in L'}\left|[p_n/q_n]_{f,\zeta}(z)-f(z)\right|\to 0$, *as* $n\to+\infty$.

The set of all such functions $f\in\mathcal{O}(\Omega)$ *is dense and* G_δ *in* $\mathcal{O}(\Omega)$.

Theorem 12 ([50]). *Let* $\mathfrak{F}$ *be a subset of* $\mathbb{N}\times\mathbb{N}$ *containing a sequence* $(\tilde{p}_n,\tilde{q}_n)\in\mathfrak{F}$, $n=1,2,\ldots$, *with* $\tilde{p}_n\to+\infty$.*Let* $\Omega\subset\mathbb{C}$ *be a simply connected open set and* $\zeta\in\Omega$ *be fixed. Then there exists a holomorphic function* $f\in\mathcal{O}(\Omega)$, *such that, for every polynomial* h *and every compact set* $K\subset\mathbb{C}\setminus\Omega$ *with connected complement, there exists a sequence*$(p_n,q_n)\in F$, $n=1,2,\ldots$, *such that the following hold:*

- $f\in\mathfrak{D}_{p_n,q_n}(\zeta)$ *for all* $n\in N$,
- *for every compact set* $L'\subset\Omega$, *there exists a* $n(L')\in\mathbb{N}$ *such that*
  $$f\in E_{p_n,q_n,\zeta}(L'\cup K) \text{ for all } n\geq n(L'),$$
- $sup_{z\in K}\left|[p_n/q_n]_{f,\zeta}(z)-h(z)\right|\to 0$, *as* $n\to+\infty$ *and*
- $sup_{z\in L'}\left|[p_n/q_n]_{f,\zeta}(z)-f(z)\right|\to 0$, *as*$n\to+\infty$.

The set of all such functions $f\in\mathcal{O}(\Omega)$ *is dense and* G_δ *in* $\mathcal{O}(\Omega)$.

Theorem 13 ([50]). *Let* $\mathfrak{F}$ *be a subset of* $\mathbb{N}\times\mathbb{N}$ *containing a sequence* $(\tilde{p}_n,\tilde{q}_n)\in\mathfrak{F}$, $n=1,2,\ldots$, *with* $\tilde{p}_n\to+\infty$. *Let also* $\Omega\subset\mathbb{C}$ *be a simply connected open set. Then there exists a holomorphic function* $f\in\mathcal{O}(\Omega)$ *satisfying the following.*

For every compact set $K \subset \mathbb{C} \setminus \Omega$ with connected complement and every polynomial h, there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that the following hold:

For every compact set $L \subset \Omega$, there exists a $n(L) \in \mathbb{N}$ such that

- $f \in \mathfrak{D}_{p_n,q_n}(\zeta)$ *for all* $n \in \mathbb{N}$ *and* $\zeta \in L$,
- $f \in \mathcal{E}_{p_n,q_n,\zeta}(L \cup K)$ *for all* $n \geq n(L)$ *and* $\zeta \in L$,
- $sup_{\zeta \in L, z \in K}\left|[p_n/q_n]_{f,\zeta}(z) - h(z)\right| \to 0$, *as* $n \to +\infty$ *and*
- $sup_{\zeta \in L, z \in L'}\left|[p_n/q_n]_{f,\zeta}(z) - f(z)\right| \to 0$, *as* $n \to +\infty$.
- *The set of all such functions* $f \in \mathcal{O}(\Omega)$ *is dense and* G_δ *in* $\mathcal{O}(\Omega)$.

Theorem 14 ([50]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$. Let $\Omega \subset \mathbb{C}$ be as above and $L \subset \Omega$ and $L' \subset \bar{\Omega}$ two compact subsets with connected complements, such that $K \cap \bar{\Omega} = \emptyset$. Then there exists a $f \in A(\Omega) := \{g: K \to \mathbb{C}: g \text{ continuous on } \bar{\Omega} \wedge \text{holomorphic on } \Omega\}$ such that, for every polynomial p there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that:*

- $f \in \mathfrak{D}_{p_n,q_n}(\zeta)$ *and* $f \in \mathcal{E}_{p_n,q_n,\zeta}(L' \cup K)$ *for every* $n \in \mathbb{N}$ *and* $\zeta \in L$,
- $sup_{\zeta \in L, z \in K}\left|[p_n/q_n]_{f,\zeta}(z) - p(z)\right| \to 0$, *as* $n \to +\infty$ *and*
- $sup_{\zeta \in L, z \in L'}\left|[p_n/q_n]_{f,\zeta}(z) - f(z)\right| \to 0$, *as* $n \to +\infty$.

The set of all such functions $f \in A(\Omega)$ is dense and G_δ in $A(\Omega)$.

Theorem 15 ([50]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$. Let Ω be a domain in $\mathbb{C}$, such that $\{\infty\} \cup (\mathbb{C} \setminus \bar{\Omega})$ is connected and $\bar{\Omega}^o = \Omega$. Let ζ_0 be fixed. Then there exists a holomorphic function $f \in A(\Omega)$, such that, for every polynomial p and any compact set $K \subset \mathbb{C} \setminus \bar{\Omega}$ with connected complement, there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, so that*

- $f \in D_{p_n,q_n}(\zeta_0)$ *for all* $n \in \mathbb{N}$,
- $sup_{z \in K}\left|[p_n/q_n]_{f,\zeta_0}(z) - p(z)\right| \to 0$, *as* $n \to +\infty$ *and*
- *for every compact set* $L' \subset \Omega$, *there exists a* $n(L') \in \mathbb{N}$ *such that*
    - $f \in E_{p_n,q_n,\zeta_0}(L' \cup K)$ *for all* $n \geq n(L')$ *and*
    - $sup_{z \in L'}\left|[p_n/q_n]_{f,\zeta_0}(z) - f(z)\right| \to 0$, *as* $n \to +\infty$.

The set of all such functions $f \in A(\Omega)$ is dense and G_δ in $A(\Omega)$.

Theorem 16 ([50]). *Let $\mathfrak{F}$ be a subset of $\mathbb{N} \times \mathbb{N}$ containing a sequence $(\tilde{p}_n, \tilde{q}_n) \in \mathfrak{F}$, $n = 1,2, \dots$, with $\tilde{p}_n \to +\infty$. Let $\Omega \subset \mathbb{C}$ be a domain, such that $\{\infty\} \cup (\mathbb{C} \setminus \bar{\Omega})$ is connected and $\bar{\Omega}^o = \Omega$. Then there exists a holomorphic function $f \in A(\Omega)$, satisfying the following two conditions.*

- *For every compact set $K \subset \mathbb{C} \setminus \bar{\Omega}$ with connected complement and every polynomial P, there exists a sequence $(p_n, q_n) \in \mathfrak{F}$, $n = 1,2, \dots$, such that:*
    - $f \in \mathfrak{D}_{p_n,q_n}(\zeta)$ *for all* $n \in N$ *and* $\zeta \in L$,
    - $f \in \mathcal{E}_{p_n,q_n,\zeta}(L \cup K)$ *for all* $n \geq n(L)$ *and* $\zeta \in L$,
    - $sup_{\zeta \in L, z \in K}\left|[p_n/q_n]_{f,\zeta}(z) - P(z)\right| \to 0$, *as* $n \to +\infty$.
- *For every compact set* $L' \subset \bar{\Omega}$, *we have*
    - $sup_{\zeta \in L, z \in K}\left|[p_n/q_n]_{f,\zeta}(z) - f(z)\right| \to 0$, *as* $n \to +\infty$.
- *The set of all such functions* $f \in A(\Omega)$ *is dense and* G_δ *in* $A(\Omega)$.

Finally, in the paper «*Universal series on convex subsets of* $\mathbb{C}^n$» ([40]), we give an abstract introductory framework for a *theory of universal series in several variables*, from which we deduce generalizations in $\mathbb{C}^n$ for existing universality results in the complex plane.

It appears that many arguments in the one variable universality theory use Taylor power series expansions of holomorphic functions into open disks. But, in several variables, the open polydisks do not qualify to be the general target domains because of the failure of the property to be maximal domains of convergence of multiple series. Further, and more important, for $n > 1$, the index set $\mathbb{C}^n$ does not carry any natural ordering, so there is no canonical way to consider multiple series as sequences of finite partial sums as in case $n = 1$. Since, because of all these reasons, many of the most highly appreciated theorems and applications on universal series have no obvious analogue in several complex variables, one might expect that the theory of universal series in C^n lacks the appeal of the classical one variable theory.

In 2008, R. Clouâtre established the existence of power series in $\mathbb{C}^n$with the property that the subsequences of the sequence of partial sums uniformly approach any holomorphic function on any well chosen compact subset outside the set of convergence of the series[8]. The principal aim of [40] is to *propose generalizations and applications for the abstract theory of universal series in several variables*[9] and to *show how multidimensional existence proofs can be cleared of their dependence on polydisks and partial sums and reconnected to original complex analytic ideas, so that a systematic application of Baire's theorem and well known approximation theorems (such as Runge's theorem and Weierstrass' theorem) suffice to investigate series universality in several complex variables*. In this direction, the proofs of the main results in the paper need to exhibit a function approximating a given function into the space where the (universal) multiple series should live and approaching a second given function in the space where the universality property holds.

The first part of our paper is devoted to the introduction of two parallel techniques for defining and studying universal multiple series. As a particular consequence, we reprove Seleznev's theorem[10] generalization in $\mathbb{C}^n$.

The second part contains the development of a universal power series theory in several variables. The situation is different from that of the first part, due to the applicability of expansion centres for multiple Taylor series which implies radical changes to the foundation of a concise and reliable multi-dimensional theory for universal power series.

- **Summability transforms and analytic continuation in $\mathbb{C}^n$**

The second specialization of my work on Complex Analysis is determined by the investigation of counterexamples and results on *summability transforms and analytic continuation in* $\mathbb{C}^n$.

Let f be a function holomorphic in $0 \in \mathbb{C}^n$. Suppose we know its Taylor power series expansion around the origin

$$S_f(z_1, \dots, z_n) = \sum_{\nu_1,\dots,\nu_n=0}^{\infty} a_{\nu_1,\dots,\nu_n}^{(f)} z_1^{\nu_1} \dots z_n^{\nu_n} \; \left(a_{\nu_1,\dots,\nu_n}^{(f)} := \frac{1}{\nu_1! \dots \nu_n!} \frac{\partial^{\nu_1+\dots+\nu_n} f}{\partial z_1^{\nu_1} \dots \partial z_n^{\nu_n}}\bigg|_{z_1=0,\dots,z_n=0}\right).$$

If D_f is the domain of convergence of $S_f(z)$, we are interested in computing $f(z)$ for $z \notin D_f$, if it exists, in terms of the partial sums of the series $S_f(z)$.

8 See R. Clouâtre: *Universal power series in* $\mathbb{C}^N$, Canad. J. Math. Volume 54, Issue 2, 2011 pp. 230-236

9 See F. Bayart, K.G. Grosse-Erdmann, V. Nestoridis and C. Papadimitropoulos: *Abstract Theory of Universal Series and Applications*, Proc. London Math. Soc., Volume 96, Issue 3, 2008, pp. 417-463

10 See A. I. Seleznev: *On universal power series* (Russian), Mat.Sbornik (N.S.), Volume 28, 1951, pp. 453-460

If $n = 1$, the application of matrix summability methods is a classical tool of analytic continuation and in this connection the Okada theorem (see, for instance, R. E. Powell and S. M. Shah: *Summability Theory and its Applications*, Van Nostrand Reinhold, London, 1972) plays an important role: it describes a domain into which a given summability method sums an arbitrary power series to its analytic continuation, if such a domain is known for the geometric series. This theorem is often applied in the literature because, for many summability methods, it is easy to find a set into which the geometric series is summed to $\prod_{j=1}^{n}(1-z_j)^{-1}$.

Let us recall Okada's classical theorem [11]. *Suppose* $\mathcal{M} = (\pi_{m.k})_{m\geq 0;0\leq k\leq m}$ *is an infinite matrix of complex entries. Set* $\varpi(\mathcal{M}) = \{\ z \in \mathbb{C}\colon lim_{m\to\infty} \sum_{k=0}^{\infty} \pi_{m.k} \sum_{\nu=0}^{k} z^{\nu} = (1-z)^{-1}\}$ *and assume that*

- $D \subset \varpi(\mathcal{M})$ (D *is the open unit disk*),
- *the set* $\varpi(\mathcal{M})$ *is open and*
- $lim_{m\to\infty} \sum_{k=0}^{\infty} \pi_{m.k} \sum_{\nu=0}^{k} z^{\nu} = (1-z)^{-1}$ *uniformly on compact subsets of* $\varpi(\mathcal{M})$.

Put

$$\boldsymbol{g}(\varpi(\mathcal{M}))\colon = \{z \in \mathbb{C}\colon (1/z) \notin \varpi(\mathcal{M})\} \cup \{0\}.$$

Let Ω *be an open domain in* $\mathbb{C}$*, such that* $0 \in \Omega$*. If*

$$\mathbb{E}_{\mathcal{M}}(\mathcal{O}(\Omega))\colon = \{\ z \in \mathbb{C}\colon lim_{m\to\infty} \sum_{k=0}^{\infty} \pi_{m.k} \sum_{\nu=0}^{k} a_{\nu}^{(f)} z^{\nu} = f(z), whenever f \in \mathcal{O}(\Omega)\},$$

then

- $\mathbb{E}_{\mathcal{M}}(\mathcal{O}(\Omega)) = \bigcap_{z\notin\Omega} z\varpi(M) = \{z \in C\colon z\,\boldsymbol{g}(\varpi(\mathcal{M})) \subseteq \Omega\}$ *and*
- $lim_{m\to\infty} \sum_{k=0}^{\infty} \pi_{m.k} \sum_{\nu=0}^{k} a_{\nu}^{(f)} z^{\nu} = f(z)$ *uniformly on compact subsets of* $\mathbb{E}_{\mathcal{M}}(\mathcal{O}(\Omega))$.

In my thesis «Domaine de Convergence d'une Transformation de la Suite des Sommes Partielles d'une Fonction Holomorphe et Applications aux Approximants de Type Padé» ([1]) and the paper «*The convergenceof Padé-type approximants to holomorphic functions of several complex variables*» ([2]), we obtain an extension of Okada's theorem in the case of an open polydisk of $\overline{\mathbb{C}}^n$. Let us recall the relevant results. Let Ω be any open set in $\mathbb{C}^n$, $0 \in \mathbb{C}^n$, and $(n+1)$ infinite triangular matrices of complex-valued functions

$$\mathcal{N}(z) = \left(\sigma_{m,k}(z)\right)_{m\geq 0;0\leq k\leq m}, \mathcal{N}_1(z) = \left(\sigma_{m,k}^{(1)}(z)\right)_{m\geq 0;0\leq k\leq m}, \ldots, \mathcal{N}_n(z) = \left(\sigma_{m,k}^{(n)}(z)\right)_{m\geq 0;0\leq k\leq m}$$

$(z \in \mathbb{C}^n)$. The $N(z)$ –transform of the sequence of partial sums of a holomorphic function $f \in O(\Omega)$, around the origin, is the sequence

$$\left\{\sum_{k=0}^{m} \sigma_{m,k}(z) \sum_{\nu_1,\ldots,\nu_n=0}^{k} a_{\nu_1,\ldots,\nu_n}^{(f)} z_1^{\nu_1} \ldots z_n^{\nu_n}\colon z = (z_1,\ldots,z_n) \in \mathbb{C}^n, m = 0,1,2,\ldots\right\}.$$

The $(\mathcal{N}_1(z),\ldots,\mathcal{N}_n(z))$ –transform of the sequence of partial sums of a holomorphic function $f \in \mathcal{O}(\Omega)$, around the origin, is the sequence

$$\Big\{\sum_{k_1=0}^{m_1} \sigma_{m_1,k_1}^{(1)}(z) \cdot \sum_{\nu_1=0}^{k_1}\left(\sum_{k_2=0}^{m_2} \sigma_{m_2,k_2}^{(2)}(z) \cdot \sum_{\nu_2=0}^{k_2}\left(\ldots\left(\sum_{k_n=0}^{m_n} \sigma_{m_n,k_n}^{(n)}(z) \sum_{\nu_n=0}^{k_n} a_{\nu_1,\ldots,\nu_n}^{(f)} z_1^{\nu_1} \ldots z_n^{\nu_n}\right)\ldots\right)\right)\colon$$

$$z = (z_1,\ldots,z_n) \in \mathbb{C}^n, m_1 = 0,1,2,\ldots,\ldots, m_n = 0,1,2,\ldots\Big\}.$$

Furthermore, let

$\mathbb{P}_{\mathcal{N}(z)}^{\Omega}\colon = \{P' \subseteq \Omega$: for any $f \in \mathcal{O}(\Omega)$, the $\mathcal{N}(z)$ –transform of the sequence of the partial sums of f, around the origin, converges to $f(z)$, uniformly on any compact subset of P', if $m \to \infty\}$

[11] See Y. Okada: *Uber die Annaherung analytisher Funktionen*, Math. Z., 23(1925), 62-71.

$\mathbb{P}^{\Omega}_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)} := \{P'' \subseteq \Omega$: for any $f \in \mathcal{O}(\Omega)$, the $(\mathcal{N}_1(z), \dots, \mathcal{N}_n(z))$ −transform of the sequence of the partial sums of f, around the origin, converges to $f(z)$, uniformly on any compact subset of P'', if $m_{\lambda_1} \to \infty, \dots, m_{\lambda_\mu} \to \infty\}$.

We now introduce the notation to be used. We set

$$\mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big) := \cup_{P' \in \mathbb{P}^{\Omega}_{\mathcal{N}(z)}} P' \text{ and } \mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big) := \cup_{P'' \in \mathbb{P}^{\Omega}_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}} P''.$$

Next, we consider two sequences

$$\{p_m(x;z) = p_m(x_1, \dots, x_n; z_1, \dots, z_n): m = 0,1,2, \dots\} \text{ and}$$

$$\{q_{m_1,\dots,m_n}(x;z) = q_{m_1,\dots,m_n}(x_1, \dots, x_n; z_1, \dots, z_n): m_1 = 0,1,2, \dots, \dots, m_n = 0,1,2, \dots\}$$

of functions of the $2n$ complex variables $x_1, \dots, x_n, z_1, \dots, z_n$, which have the form

$$p_m(x;z) = \sum_{k=0}^{m} \sigma_{m,k}(z) \sum_{\nu_1,\dots,\nu_n=0}^{k} x_1^{\nu_1} \dots x_n^{\nu_n} z_1^{\nu_1} \dots z_n^{\nu_n}$$

and

$$q_{m_1,\dots,m_n}(x;z) = \sum_{k_1=0}^{m_1} \sigma^1_{m_1,k_1}(z) \cdot \sum_{\nu_1=0}^{k_1} \Big(\sum_{k_2=0}^{m_2} \sigma^2_{m_2,k_2}(z) \cdot \sum_{\nu_2=0}^{k_2} \Big(\dots \Big(\sum_{k_n=0}^{m_n} \sigma^n_{m_n,k_n}(z) \sum_{\nu_n=0}^{k_n} x_1^{\nu_1} \dots x_n^{\nu_n} z_1^{\nu_1} \dots z_n^{\nu_n} \Big) \dots \Big)\Big).$$

Suppose that

$\omega' = \omega'\big(\mathcal{N}(z)\big)$ is the maximal open set in $\mathbb{C}^{2n}$ into which the functions $p_m(x;z)$ are continuous and the sequence $\{p_m(x;z): m = 0,1,2, \dots\}$ converges uniformly on every compact subset to $\prod_{j=1}^{n}\big(1 - x_j z_j\big)^{-1}$.

Further, assume that

$\omega'' = \omega''\big(\mathcal{N}_1(z), \dots, \mathcal{N}_n(z)\big)$ is the maximal open set in $\mathbb{C}^{2n}$ in which the functions $q_{m_1,\dots,m_n}(x;z)$ are continuous and the sequence $\{p_m(x;z): m = 0,1,2, \dots\}$ converges uniformly on every compact subset to $\prod_{j=1}^{n}\big(1 - x_j z_j\big)^{-1}$, if $m_{\lambda_1} \to \infty, \dots, m_{\lambda_\mu} \to \infty$.

For $\omega = \omega', \omega''$, we set

$$\boldsymbol{g}(\omega;\Omega) := \Big\{ z = (z_1, \dots, z_n) \in \mathbb{C}^n : \Big(\frac{1}{\zeta_1}, \dots, \frac{1}{\zeta_n}, z_1, \dots, z_n\Big) \in \omega, \forall \zeta_j \in \bar{\mathbb{C}} \setminus pr_j(\Omega), j = 1, \dots, n \Big\}.$$

Notice that, in general, $\boldsymbol{g}(\omega;\Omega) \not\subset \Omega$ $(\omega = \omega', \omega'')$. In fact, it suffices to take

$$\Omega = \{(z_1, z_2) \in \mathbb{C}^2 : 0 < |z_1| < |z_2| < 1\} \cup \{(z_1, z_2) \in \mathbb{C}^2 : |z_1| < 1, |z_2| < 1\} \text{ and}$$
$$M = (\pi_{m.k})_{m \geq 0; 0 \leq k \leq m} = (\delta_{m.k})_{m \geq 0; 0 \leq k \leq m}$$

where $\delta_{m.k}$ is the Krönecker symbol ([1], [4]).

We can obtain the following.

Theorem 17 ([1], [2]).If Ω is a polydisk in $\mathbb{C}^n$ or $\bar{\mathbb{C}}^n$, then

$$\boldsymbol{g}(\omega';\Omega) \subset \mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big) \text{ and } \boldsymbol{g}(\omega'';\Omega) \subset \mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big).$$

Our result generalizes the Gawronski-Trautner extension of Okada's theorem[12]: *If* $\Omega \subsetneq \mathbb{C}$, $\mathcal{M} = \left(\pi_{m,k} \in \mathbb{C}\right)_{m\geq 0; 0\leq k\leq m}$ *and the sequence* $\left(\sum_{k=0}^{\infty} \pi_{m.k} \sum_{\nu=0}^{k} z^{\nu} : z \in \mathbb{C}, m = 0,1,2, \dots\right)$ *converges uniformly on every compact subset of an open* $w \subset \mathbb{C}$*, containing* 0*, to* $(1-z)^{-1}$*, then*

$$\bigcap_{\zeta \in \mathbb{C}\setminus\Omega} \zeta w \subset \mathbb{E}^1_{\mathcal{M}}\big(\mathcal{O}(\Omega)\big).$$

Two natural questions which may be asked are the following.

- Is the domain $\boldsymbol{g}(\omega'; \Omega)$ always contained in $\mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big)$?
- Is the domain $\boldsymbol{g}(\omega''; \Omega)$ always contained in $\mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big)$?

In the paper «*Two counterexamples for the Okada Theorem in* $\mathbb{C}^n$» ([4]), we give two counter-examples which show that the answers are negative Let's be more specific.

Counter-example 1 ([4]).*Let* Ω *be an open subset of* $\mathbb{C}^n$, $0 \in \Omega$, *such that*

$pr_j(\Omega) = D^n := \{z = (z_1, \dots, z_n) \in \mathbb{C}^n : |z_1| < 1, \dots, |z_n| < 1\} j = 1,2,\dots,n$ *and*

(Ω, D^n) *is not a Runge pair.*

Then there are $(n+1)$ *infinite triangular matrices*

$\mathcal{N}(z) = \left(\sigma_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}$, $\mathcal{N}_1(z) = \left(\sigma^1_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}$, …, $\mathcal{N}_n(z) = \left(\sigma^n_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}$

satisfying $\boldsymbol{g}(\omega'; \Omega) \not\subseteq \mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big)$ and $\boldsymbol{g}(\omega''; \Omega) \not\subseteq \mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big)$.

Counter-example 2 ([4]).*Let*

$$\Omega = \{z = (z_1, z_2) \in \mathbb{C}^2 : |z_1| < 1, |z_2| < 1, \}|z_1 + z_2| < 1.$$

Then there are three infinite triangular matrices

$$N(z) = \left(\sigma_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}, N_1(z) = \left(\sigma^1_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}, N_2(z) = \left(\sigma^2_{m,k}(z)\right)_{m\geq 0; 0\leq k\leq m}$$

such that

- *for* $f : \Omega \to \mathbb{C} : (z_1, z_2) \longmapsto f(z_1, z_2) = \frac{1}{1-(z_1+z_2)} \in \mathcal{O}(\Omega)$ *and*
- *for* $z_1 > 0$ *suitably choosen and near to* 1*,*

the point $(z_1, -z_1)$ *satisfies*

- $f(z_1, -z_1) \neq \sum_{k=0}^{m} \sigma_{m,k}(z) \sum_{\nu_1,\nu_2=0}^{k} a^{(f)}_{\nu_1,\nu_2} z_1^{\nu_1} z_2^{\nu_2}$
  and
- $f(z_1, -z_1) \neq \sum_{k_1=0}^{m_1} \sigma^1_{m_1,k_1}(z) \cdot \sum_{\nu_1=0}^{k_1} \left(\sum_{k_2=0}^{m_2} \sigma^2_{m_2,k_2}(z) \sum_{\nu_2=0}^{k_2} a^{(f)}_{\nu_1,\nu_2} z_1^{\nu_1} z_2^{\nu_2}\right)$.

It follows that

$$\boldsymbol{g}(\omega'; \Omega) \not\subseteq \mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big) \text{ and } \boldsymbol{g}(\omega''; \Omega) \not\subseteq \mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big).$$

Further, in [4], we modify the form of the sets $\boldsymbol{g}(\omega; \Omega)$ $(\omega = \omega', \omega'')$ and we construct two new domains

$$\mathcal{G}(\omega'; \Omega) = \mathcal{G}\big(\omega'(\mathcal{N}(z)); \Omega\big) \text{ and } \mathcal{G}(\omega''; \Omega) = \mathcal{G}\big(\omega''(\mathcal{N}_1(z), \dots, \mathcal{N}_n(z)); \Omega\big)$$

which are contained in

$$\mathbb{E}^n_{\mathcal{N}(z)}\big(\mathcal{O}(\Omega)\big) \text{ and } \mathbb{E}^n_{\mathcal{N}_1(z),\dots,\mathcal{N}_n(z)}\big(\mathcal{O}(\Omega)\big)$$

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12 See W. Gawronski and R. Trautner: *Verschärfung eines Satzes von Borel-Okada über Summierbarkeit von Potenzreihen*, Period. Math. Hungar. 7 (1976), 201-211

respectively, under the assumption that Ω is a polydomain, i.e. a Cartesian product of domains of $\mathbb{C}$. Specifically, we prove the following.

Theorem 18 ([4]). *Consider the family of sets*

$(S)_n^\infty := \{\Omega$ *open set in* $\mathbb{C}^n$ *such that* $0 \in \Omega$ *and, for any* $z \in \Omega$, *there is a simply connected polydomain* $U_z = U_z^{(1)} \times U_z^{(2)} \times \ldots \times U_z^{(n)}$ *such that* $\{0, z\} \subset U_z \subset\subset \Omega$ *and* $\partial U_z^{(j)}$ *is smooth* $(: C^\infty)$, $j = 1,2,\ldots,n\}$.

If $\Omega \in (S)_n^\infty$, *then there holds*

$$\boldsymbol{\mathcal{G}}(\omega';\Omega) = \boldsymbol{\mathcal{G}}(\omega'(\mathcal{N}(z));\Omega) \text{ and } \boldsymbol{\mathcal{G}}(\omega'';\Omega) = \boldsymbol{\mathcal{G}}(\omega''(\mathcal{N}_1(z),\ldots,\mathcal{N}_n(z));\Omega).$$

However, in general, a literal extension of Okada's theorem would be false in dimension more than one. Thus, given any function f holomorphic in $0 \in \mathbb{C}^n$, the problem of computing $f(z)$ abroad the domain D_f of convergence of $S_f(z)$ must be cleared of its dependence on the Okada presuppositions and reconnected to the general theory of matrix summability methods. In this direction, let us consider the $\mathbb{C}$ −linear functional Λ_f associated with f and defined uniquely on the space $\mathcal{P}(\mathbb{C}^n)$ of all holomorphic polynomials by

$$\Lambda_f\left(x_1^{\nu_1}\ldots x_n^{\nu_n}\right) = \frac{1}{\nu_1!\ldots\nu_n!}\frac{\partial^{\nu_1+\cdots+\nu_n}f}{\partial z_1^{\nu_1}\ldots\partial z_n^{\nu_n}}\bigg|_{z_1=0,\ldots,z_n=0}, \nu = (\nu_1,\ldots,\nu_n) \in \mathbb{N}^n .$$

By Cauchy's integral formula, *it holds*

$$\Lambda_f\left(\prod_{j=1}^n\left(1 - x_j z_j\right)^{-1}\right) = S_f(z) = f(z) \qquad (8)$$

pointwise in D_f. We investigate general conditions under which (8) extends to an open set Ω containing D_f. To do so, we prove the following.

Theorem 19 ([4]). *Let* Ω *be a bounded Runge domain in* $\mathbb{C}^n$ $(0 \in \Omega)$. *Let also* $\Delta^n(0;\rho) = \Delta^n(0,\ldots,0;\rho_1,\ldots,\rho_n)$ *be an open polydisk such that*

$$\Delta^n(0,\ldots,0;\rho_1,\ldots,\rho_n) \subset\subset \Omega \subset\subset \Delta^n(0,\ldots,0;\rho_1^{-1},\ldots,\rho_n^{-1}).$$

Let finally

$$V_\rho$$

be the maximal open neighborhood of $\Delta^n(0,\ldots,0;\rho_1,\ldots,\rho_n) \times \Delta^n(0,\ldots,0;\rho_1^{-1},\ldots,\rho_n^{-1})$ *into which the geometrical series* $\sum_{\nu_1,\ldots,\nu_n=0}^\infty x_1^{\nu_1}\ldots x_n^{\nu_n} z_1^{\nu_1}\ldots z_n^{\nu_n}$ *converges uniformly on every compact set to* $\prod_{j=1}^n\left(1 - x_j z_j\right)^{-1}$.

If $q_{m_1,\ldots,m_n}(x_1,\ldots,x_n,z_1,\ldots,z_n)$ *are holomorphic polynomials in* $(x_1,\ldots,x_n,z_1,\ldots,z_n)$ *such that*

$$lim_{m_1\to\infty,\ldots,m_n\to\infty} q_{m_1,\ldots,m_n}(x_1,\ldots,x_n,z_1,\ldots,z_n) = \prod_{j=1}^n\left(1 - x_j z_j\right)^{-1}$$

uniformly on every compact subset of V_ρ *and if*

$$sup_{m_j\in N(j=1,2,\ldots,n)}\left(sup_{|s_j|=d_j^{-1}(j=1,2,\ldots,n)}\left|q_{m_1,\ldots,m_n}(x_1,\ldots,x_n,z_1,\ldots,z_n)\right|\right) < \infty$$

for some $(d_1,\ldots,d_n)$ *with* $0 < d_j < \rho_j (j = 1,2,\ldots,n)$, *then, for any* $f \in \mathcal{O}(\Omega)$, *we have*

$$lim_{m_1\to\infty,\ldots,m_n\to\infty}\Lambda_f\left(q_{m_1,\ldots,m_n}(x_1,\ldots,x_n,z_1,\ldots,z_n)\right) = f(z)$$

pointwise in Ω.

Example 3 ([4]). *Letting*

$$\sigma^{j}_{m_j,k_j} = \begin{cases} t^{(j)}_{m_j} \cdot \frac{d_j^{2m_j}}{\sum_{\nu_j=0}^{m_j} d_j^{2\nu_j}}, if\, k_j \neq m_j \\ 0, otherwise \end{cases}$$

with $0 < d_1, \dots, d_n < \frac{1}{2}$*and*

$$\Delta^n(0,\dots,0; d_1,\dots,d_n) \subset\subset \Delta^n(0,\dots,0; \rho_1,\dots,\rho_n) \subset\subset \Omega$$
$$\subset\subset \Delta^n(0,\dots,0; \rho_1^{-1},\dots,\rho_n^{-1}) \subset\subset \Delta^n(0,\dots,0; d_1^{-1},\dots,d_n^{-1})$$

the polynomials

$$q_{m_1,\dots,m_n}(x_1,\dots,x_n,z_1,\dots,z_n) = \sum_{k_1=0}^{m_1} \sigma^1_{m_1,k_1}(z) \cdot$$
$$\sum_{\nu_1=0}^{k_1}\left(\sum_{k_2=0}^{m_2} \sigma^2_{m_2,k_2}(z) \cdot\right.$$
$$\left.\sum_{\nu_2=0}^{k_2}\left(\dots\left(\sum_{k_n=0}^{m_n} \sigma^n_{m_n,k_n}(z) \sum_{\nu_n=0}^{k_n} x_1^{\nu_1}\dots x_n^{\nu_n} z_1^{\nu_1}\dots z_n^{\nu_n}\right)\dots\right)\right)$$

satisfy the assumptions of theorem 19.

Biholomorphic maps in $\mathbb{C}^n$

The third specialization of my work on Complex Analysis leads to counterexamples and results on *biholomorphic maps in* $\mathbb{C}^n$. Remind that if $\Omega \subset \mathbb{C}^n$and $\Omega' \subset \mathbb{C}^n$are two open sets, then the map $F: \Omega \to \Omega'$ is *biholomorphic*,if F is a holomorphic homeomorphism with holomorphic inverse $F^{-1}: \Omega' \to \Omega$. The open sets $\Omega \subset \mathbb{C}^n$ and $\Omega' \subset \mathbb{C}^n$are called *biholomorphically equivalent* if there is a biholomorphic map$F: \Omega \to \Omega'$.

First, in the paper *«On Riemann's mapping theorem»* ([14]), we study biholomorphic inequivalence between strictly convex bounded domains in $\mathbb{C}^n$ and we propose functional theoretic approaches to the formulation problem of a Riemann mapping theorem in several complex variables.

Counter-example 4 ([14]).*The domain*

$$\Omega = \{z = (z_1, z_2, z_3) \in \mathbb{C}^3 : |z_1|^2 + |z_2|^2 + |z_3|^4 < 1\}$$

is a strictly convex domain in $\mathbb{C}^3$ *which is not biholomorhically equivalent to the unit ball of* $\mathbb{C}^3$.

Despite this negative result, we prove a very general criterion for the biholomorphic equivalence of two open pseudoconvex domains.

Theorem 20 ([14]). *Let* $\Omega \subset \mathbb{C}^n$ *and* $\Omega' \subset \mathbb{C}^n$ *be two open pseudoconvex sets. Then* Ω *and* Ω' *are biholomorphically equivalent if and only if there exists a* $\mathbb{C}-$*linear multiplicative isomorphism* $T: O(\Omega') \to O(\Omega)$.

Next, in the paper *«Generalized Padé-type approximation and integral representation»* ([17]), we study boundary regularity properties of biholomorphic maps. It is well kown that the mapping properties of the Bergman kernel function have a central role in the study of bianalytic maps. It is also well known that the statement of a *Riemann Mapping Theorem* in several complex variables must be quite different than in one variable. Chern and Moser built on the pioneering work of Poincaré and E. Cartan to produce a complete set of differential-geometric boundary invariants which must be preserved under a bianalytic map between smooth strictly pseudoconvex domains in $\mathbb{C}^n$[13]. In order to see that the Chern-Moser invariants are preserved under a bianalytic mapping, it is important to know that a bianalytic map between smooth strictly pseudoconvex domains must extend smoothly to the boundary. A fundamental result dealing with the C^∞extension to the boundary of bianalytic maps between smooth strictly pseudoconvex domains was proved in 1974 by Fefferman[14]. This result is classical in one complex variable, but in several variables it had been a major outstanding conjecture for many years. The first *Proof* in the one variable setting seems to be due to Painlevé)[15]. Other proofs were given by Kellogg and Warschawski[16]. One would like to adapt the proof of this result to more general situations, however many obstacles present themselves. The main purpose of this section is to propose an extension of *Painlevé's Theorem* in the case of arbitrary open subsets of $\mathbb{C}^n$, by using generalized Padé-type approximants.

Let $\Omega \neq \emptyset$ be a bounded open subset of $\mathbb{C}^n$. The subspace $\mathcal{O}L^2(\Omega) = \mathcal{O}(\Omega) \cap L^2(\Omega)$ is closed in the Hilbert space $L^2(\Omega)$ and hence is itself a Hilbert space. The evaluation map $\mathcal{O}L^2(\Omega) \to \mathbb{C}: f \longmapsto f(w)$ is a continuous $\mathbb{C}$ −linear functional, whenever $w \in \Omega$. By the *Riesz Representation Theorem*, there exists a unique element $K_\Omega(\cdot, w) \in \mathcal{O}L^2(\Omega)$ such that

$$f(w) = \int_\Omega f(\zeta)\overline{K_\Omega(\zeta, w)}.dV(\zeta) = \langle f, K_\Omega(\cdot, w)\rangle$$

for all $f \in \mathcal{O}L^2(\Omega)$. Here, $\langle \cdot, \cdot \rangle$ denotes the inner product in $L^2(\Omega)$and $\|\cdot\|_2$ the corresponding norm. Remind that $K_\Omega(z, w)$ is called *the Bergman kernel function in*Ωas a function of z. It is analytic in z, conjugate analytic in $w$and satisfies $K_\Omega(z, w) = \overline{K_\Omega(w, z)}$. There is a bounded orthogonal projection, of norm 1, $P_\Omega: L^2(\Omega) \to \mathcal{O}L^2(\Omega)$ called *the Bergman projection of*Ω, satisfying

$$P_\Omega(f)(\cdot) = \int_\Omega K_\Omega(\cdot, w) f(w) dV(w),\ f \in L^2(\Omega).$$

The property that the Bergman projection operator preserves differentiability up to the boundary can be used in the study of boundary regularity of bianalytic maps. *The open set* Ω *is said to satisfy condition* (R_d), for some $d \in N$, if there is an integer s with $d + s > 0$ such that the Bergman projection operator is a bounded map from $C^{d+s}(\bar{\Omega})$ into $C^d(\bar{\Omega})$, that is

$$\sum_{\substack{a \in \mathbb{N}_0^n \\ |a| \leq d}} sup_{z \in \bar{\Omega}} \left| D^a \int_\Omega K_\Omega(z, w) f(w) dV(w) \right| \leq c_d \sum_{\substack{a \in N_0^n \\ |a| \leq d+s}} sup_{z \in \bar{\Omega}} |D^a f(z)|,$$

$(f \in C^{d+s}(\bar{\Omega}))$ for some constant $c_d < \infty$. Here, we have used the notation

$$D^a = \frac{\partial^{|a|}}{\partial z_1^{a_1} \ldots \partial z_n^{a_n}}\ (a = (a_1, \ldots, a_n), |a| = a_1 + \cdots + a_n).$$

The open setΩ *is said to satisfy condition* (R), if it satisfies (R_d), for any $d \in \mathbb{N}$. One of *Bell's Theorems* says that *abianalytic map between bounded pseudoconvex domains in* $\mathbb{C}^n$ *with* C^∞ *smooth boundary extends smoothly to the boundary as soon as at least one of the domains satisfies condition* (R). Another important program in this direction has been initiated by Ligocka[17]. The single most general contribution to come from this program is the discovery that i*f* Ω *and* Ω' *are bounded pseudoconvex domains in* $\mathbb{C}^n$ *with boundaries of class* C^d *which satisfy condition* (R_d), *then every biholomorhic map of* Ω to Ω' *is in* $C^d(\bar{\Omega})$.

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13 See CHERN, S.S. and MOSER, J.K.: *Real hypersurfaces in complex manifolds*, Acta Math. 133 (1974), 219-271.

14 See FEFFERMAN, C.: *The Bergman kernel and biholomorphic mappings on pseudoconvex domains*, Invent. Math. 26 (1974), 1-65.

15 See PAINLEVÉ, P.: Sur la theorie de la representation conforme, C.R.Acad. Sciences Paris 112 (1891), 653-657.

16 See KRANTZ, S.G.: Function theory of several complex variables, John Wiley & Sons, New York, 1982and RANGE, R.M.: Holomorphic functions and integral representations in several complex variables, Springer-Verlag, 1986.

17 See LIGOSKA, E.: *The Hölder continuity of the Bergman projection and proper holomorphic mappings*, Studia Math. 80 (1984), 89-107.

Condition (R) holds for many domains of $\mathbb{C}^n$ (for example, every smoothly bounded complete Reinhardt domain satisfies condition (R)[18]. But on the other hand, Barrett found a smoothly bounded not-pseudoconvex domain in $\mathbb{C}^2$ which does not have property (R_d) for any $d \in \mathbb{N}$. It should be mentioned that, since the Bergman projection P_Ω and the $\bar{\vartheta}$ −Neumann operator N are related via *Kohn's Formula*: $P_\Omega = I - \bar{\vartheta}^* N \bar{\vartheta}$ (where $\bar{\vartheta}^*$ is the formal adjoint of the operator $\acute{\vartheta}$), whenever the $\acute{\vartheta}$ −Neumann operator associated to a domain satisfies global regularity estimates that domain satisfies condition (R). Kohn has shown that the $\bar{\vartheta}$ −Neumann operator N satisfies these estimates in a variety of domains[19]. Among these domains are smoothly bounded strictly pseudoconvex domains [20] and bounded pseudoconvex domains with real-analytic boundary [21]. In [17], we give sufficient conditions for the extension of *Painlevé's classical theorem* in $\mathbb{C}^n$. These considerations seem to be theoretical, but they succeed in eliminating both differential-geometry and subelliptic estimates for the $\acute{\vartheta}$ −Neumann problem and, on the other hand, they connect bianalytic extension problems with approximation and interpolation theory.

Let Ω be any bounded, non empty, open subset of $\mathbb{C}^n$ and let $\{\varphi_j : j = 0,1,2,\dots\}$ be an orthonormal basis for $OL^2(\Omega)$. Choose an infinite triangular matrix

$$\mathcal{M} = \left(\pi_{m,k}\right)_{m\geq 0; 0\leq k\leq m}$$

such that for any $m \geq 0$ it holds true

$$\pi_{m,k} \in \Omega(\, k \leq m),\ \pi_{m,k} \neq \pi_{m,k'}\ (k \neq k'),\ \pi_{m,k} \notin \cup_{0\leq j\leq m} Ker\bar{\varphi}_j\ (k \leq m)\ ,\ det\left[\overline{\varphi_j\left(\pi_{m,k}\right)}\right]_{k,j\neq 0} \neq 0.$$

Consider the associated generalized Padé-type approximation sequence to Bergman's projection P_Ω:

$$\left\{T_{P_\Omega(\cdot)}\left(G_m(x,\cdot)\right): m = 0,1,2,\dots\right\}.$$

In what follows, we shall assume that, whenever $w \in \Omega$ is fixed,

(C_d) *the series* $\sum_{\nu=0}^{\infty} \varphi_\nu(\cdot)\, \overline{\varphi_\nu(w)}$ *converges to* $K_\Omega(\cdot, w)$ *in* $C^d(\bar{\Omega})$;

then, it is immediately seen that $K_\Omega(\cdot, w) \in C^d(\bar{\Omega})$ and therefore $T_{P_\Omega\left(C^d(\bar{\Omega})\right)}\left(G_m(x,\cdot)\right) \subset C^d(\bar{\Omega})$. In other words, the subspace $C^d(\bar{\Omega})$ is an invariant subspace of the generalized Padé-type approximation operators. Under the general presupposition (C_d), the restriction operators

$$T_{P_\Omega(\cdot)}\left(G_m(x,\cdot)\right)\Big/_{C^d(\bar{\Omega})}$$

are continuous with respect to the topology induced by the norm $\|\cdot\|_{C^d(\bar{\Omega})}$ of $C^d(\bar{\Omega})$.

Further, we have the

Theorem 21 ([17]). *Let* $d \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$. *Assume that there isa complete orthonormal basis*$\{\varphi_\nu \in C^d(\bar{\Omega}): \nu = 0,1,2,\dots\}$ *for* $OL^2(\Omega)$*and an infinite triangular matrix*

$$\mathcal{M} = \left(\pi_{m,k}\right)_{m\geq 0; 0\leq k\leq m}$$

consisting of points $\pi_{m,k}$ *in* Ω *such that*

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[18] See RANGE, R.M.: Holomorphic functions and integral representations in several complex variables, Springer-Verlag, 1986.

[19] See KOHN, J.J.: *Global regularity for* $\acute{\vartheta}$ *on weakly pseudoconvex manifolds*, Trans. Amer. Math. Soc. 181 (1973), 273-292.

[20] See GRAUERT, H. and REMMERT, R.: Theory of Stein spaces, Transl. by A. Huckleberry, Springer-Verlag, New York, 1979.

[21] See KOHN, J.J.: *Subellipticity of the -Neumann problem on pseudoconvex domains: sufficient conditions*, Acta Math. 142 (1979), 79-122 and RANGE, R.M.: Holomorphic functions and integral representations in several complex variables, Springer-Verlag, 1986.

$\pi_{m,k} \neq \pi_{m,k'}$ $(k \neq k')$, $\pi_{m,k} \notin \cup_{0 \leq j \leq m} Ker\bar{\varphi}_j$ $(k \leq m)$, $det\left[\overline{\varphi_j(\pi_{m,k})}\right]_{k,j\neq 0} \neq 0$

and

$$lim_{m\to\infty} \sum_{j=0}^{m} \varphi_j(x) \left[\sum_{k=0}^{m} \frac{\varphi_\nu(\pi_{m,k})}{\varphi_j(\pi_{m,k})}\right] = \varphi_\nu(x)$$

for every $x \in \Omega$ *and every* $\nu = 0,1,2,\ldots$ *Then,* Ω *satisfies condition* (C_d).

Integral representations in $\mathbb{C}$ and $\mathbb{C}^n$

In the paper *«Integral representations for Padé-type operators»* ([17]), we prove that *if, in particular, the periodic function is of class* L^2 *(or harmonic) and the interpolation points lie in* D*, then one can construct integral representations for the associated Padé-typeapproximants.* With the help oftheserepresentations,we obtain explicit forms for the Padé-type operators. These are integral operators associating a Padé -type approximant to each L^2 (or harmonic) function.Their usefulness lies intheir applicabilityto the convergence problem for a series of Padé-type approximants and the identification of theset of allperiodicfunctionswiththe set of all the associated Padé-typeapproximants.

In several complex variables, the multivariate rational approximation theory is based on the polynomial interpolation of the multidimensional Cauchy kernel and leads to complicated computations. In the paper *«Generalized Padé-type approximation and integral representation»* ([29]), we replace the multidimensional Cauchy kernel $\prod_{j=1}^{n}(1 - x_j z_j)^{-1}$ by the Bergman kernel function $K_\Omega(z,x)$ into an open bounded subset Ω of $\mathbb{C}^n$ and, by using interpolating polynomials for $K_\Omega(z,x)$, we define approximants $(GPTA/m)_f(z)$ to any $f(z)$ in the space $OL^2(\Omega)$ of all holomorphic functions on Ω which are of class L^2. The characteristic property of such an approximant is that its Fourier series representation with respect to an orthonormal basis for $OL^2(\Omega)$ matches the Fourier series expanstion of f as far as possible. After studying error formulas and the convergence problem, we show that these approximants have integral representations which give rise to the consideration of an integral operator which maps every $f \in OL^2(\Omega)$ to an approximant $(GPTA/m)_f$ to f. By the continuity of this operator, we obtain some convergence results about series of analytic functions of class L^2. Our study concludes with the extension of these ideas into every functional Hilbert space.

Writings

Besidesthe above specifications,my work on the generalfield of Pure Mathematics includes six (6) teaching monographs.

The monograph entitled «Géométrie de Contact et Equations Différentielles Non-Linéaires d' Ordre 2» ([95]) refersto the implementations of contact geometry to the problem of solving non-linear differentialequations. The monograph «Topologie et Calcul Différentiel» ([96]) lays the necessary ground work for developing a theory of differential calculus in one variable to be fair generalization to th ecase of several variables. Further, in «Complex Analysis» ([99]) and «Theory of Complex Functions. Issue 1: Geometric Analysis of the Complex Plan» ([100]), we are laying the fundamentals of foundation of a modern geometric theory of functions of complex analysis. Finally, in «Projective and Descriptive Geometry» ([101]), we collect basic principles for a first descriptive geometry algorithms approach, while in «Mathematical Analysis. Issue 1: Introduction» ([102]), we explain the elementary methods for an informative undergraduate course of Mathematical Analysis.

History of continued fractions. Connection to rational approximation theory

In the paper «*Padé and Padé-type approximation for a 2π-periodic* L^p*–function*» ([9]), we give an overview of the long history of continued fractions and their connection to rational approximation theory.Below, we will develop in a fewlines,this historicaloverview.

One of the strong motivations for studying rational approximations is the perennial and concrete problem of representing functions efficiently by easily computed expressions. In this capacity the rational functions $R(x) = \sum_{\nu=0}^{n} a_\nu x^\nu / \sum_{\nu=0}^{m} b_\nu x^\nu$ have been found to be extremely effective. In a loose manner of speaking, one may say that the curve-fitting ability of $R(x)$ is roughly equal to that of a polynomial of degree $n+m$. However, in competing with the polynomial of degree$n+m$, $R(x)$has an unsuspected advantage in that the computation of $R(x)$ for a given x does not require $n+m$ additions, $n+m-1$ multiplications, and one division as might be surmised at first. By transforming $R(x)$ into a continued fraction

(CF) $$R(x) = P_1(x) + \frac{c_2|}{|P_2(x)} + \frac{c_3|}{|P_3(x)} + \cdots + \frac{c_k|}{|P_k(x)}$$

(in which each P_j denotes a certain polynomial), we achieve the significant reduction in the number of *«long»* arithmetic operations (multiplications and divisions) to n or m. In fact, in [16], we prove that *any rational function* $R(x)$ *can be put into the continued fraction form*(CF), *and from this it can be evaluated for any*x *with at most*$max\{n,m\}$*long operations*.

A few historical comments might now help to put matters in perspective. Our principal sources of information are Brezinski's precious papers: *The long history of continued fractions and Padé approximants* (in "Padé approximation and its applications. Amsterdam 1980", M.G. de Bruin and H. Van Rossum eds., Lectures Notes in Mathematics 888, Springer Verlag, Heidelberg, 1981), and *The birth and early developments of Padé approximants* (presented at the 86th summer meeting of the American Mathematical Society, Toronto, August 23-27, 1982).

The first use of continued fractions goes back to the algorithm of Euclid (c.306 B.C.-c. 283 B.C.) for computing the g. c. d. of two positive integers which leads to a terminating continued fraction. Euclid's algorithm is related to the approximate simplification of ratios as it was practiced by Archimedes (287 B.C.-212 B.C.) and Aristarchus of Samos (c.310 B.C.-c.230 B.C.). Continued fractions were also implicitly used by Greek mathematicians, such as Theon of Alexandria (c.365 B. C.), in methods for computing the side of a square with a given area. Another very ancient problem which also leads to the early use of continued fractions is the problem of the diophantine equations in honor to Diophantus (c.250 A.D.) who found a rational solution of the equation $ax \pm by = c$, where a,b and c are given positive integers. This problem has been completely solved by the Indian mathematician Aryabhata (475-550), who wrote down explicitly the continued fraction fora/b. Around 1150, one of the most important Indian mathematicians, Bhascara, wrote a book "*Líláváti*", where he treated the equation$ax - by = c$. He proved that the solution can be obtained from the continued fraction for a/b. He also showed that the convergents $C_k = A_k/B_k$of this continued fraction satisfy:$A_k = q_k A_{k-1} + A_{k-2}$, $B_k = q_k B_{k-1} + B_{k-2}$ and $A_k B_{k-1} - A_{k-1} B_k = (-1)^{k-1}$. Then the solution is given by $x = \mp c B_{n-1} + bt$ and $y = \mp c A_{n-1} + at$, according as a $B_{n-1} - b A_{n-1} = \pm 1$.

In Europe, the birth place of continued fractions is the north of Italy. The first attempt for a general definition of a continued fraction was made by Leonardo Fibonacci (c.1170-c. 1250). In his book "*Liber Abaci*" (written in 1202, revised in 1228 but only published in 1857), he introduced a kind of ascending continued fraction which is not of great interest. The first mathematician who really used our modern infinite continued fractions was Rafael Bombielli

(1526-1572) the discoverer of imaginary numbers. In his book "*L' Algebra Opera*", published in 1579 in Bologna, he gave a recursive algorithm for extracting the square root of 13 which is completely equivalent to the infinite continued fraction

$$\sqrt{13} = 3 + \frac{4|}{|6} + \frac{4|}{|6} + \cdots$$

has been introduced in 1898 by Alfred Pringsheim (1850-1941)). The next and most important contribution to the theory of continued fractions is by Pietro Antonio Cataldi (1548-1626) who can be considered as the real founded of the theory. In his book "*Trattato del modo brevissimo di trovare la radice quadra delli numeri...*" published in Bologna in 1613, he followed the same method as Bombielli for extracting the square root and he was the first to introduce a symbolism for continued fractions. He computed the continued fraction for$\sqrt{18}$ up to the 15th convergent and proved that the convergents are alternatively greater and smaller than $\sqrt{18}$ and that they converge to it. The words "*continued fractions*" were invented in 1655, by the English mathematician John Wallis (1616-1703), in his book "*Arithmetica Infinitorum*", where the author gave for the first time our modern recurrence relationship for the convergents of a continued fraction. We also mention the Dutch mathematician and astronomer Christiaan Huygens (1629-1695) who built, in 1682, an automatic planetarium. He used continued fractions for this purpose as described in his book "*Descriptio automati planetari*" published after his death.

The major contribution to the theory of continued fractions is due to Leonhard Euler (1707-1783). In his first paper on the subject, dated 1737, he proved that every rational number can be developed into a terminating continued fraction, that an irrational number gives rise to an infinite continued fraction and that a periodic continued fraction is the root of a quadratic equation. He also gave the continued fractions for e, $(e+1)/(e-1)$, $(e+1)/2$ by integrating the Riccatti equation by two different methods. Apart from the convergence of these continued fractions which he did not treated, Euler proved the irrationality of e and e^2. Euler's celebrated book "*Introductio in analysis infinitorum*", published in Lausanne in 1748, contains the first extensive and systematic exposition of the theory of continued fractions. In chapter 18, he gives the recurrence relationship for the convergents $C_k = A_k/B_k$ of the continued fraction

$$b_0 + \frac{a_1|}{|b_1} + \frac{a_2|}{|b_2} + \cdots$$

and then shows how to transform a continued fraction into a series $C_n - C_{n-1} = (-1)^{n-1} a_1 \dots a_n/B_1 \dots B_n$. This leads to the relation $C = b_0 + \sum_{n=1}^{\infty}(-1)^n a_1 \dots a_n/B_1 \dots B_n$. Reciprocally, Euler shows that an infinite series can be transformed into a continued fraction $\sum_{n=1}^{\infty}(-1)^{n-1}C_n = C_1 + \frac{C_2|}{|C_1-C_2} + \cdots + \frac{C_{n-2}C_n|}{|C_{n-1}-C_n} + \cdots$ After some examples, he treats the case of a power series. Then, he comes to the problem of convergence showing how to compute the value of the periodic continued fraction $C + \frac{1|}{|2} + \frac{1|}{|2} + \cdots$ by writing $C = 1/(2+C)$which gives $C^2 + 2C = 1$ and thus $C = \sqrt{2} - 1$. From this example he derives Bombielli' s method for the continued fraction expansion of the square root and a general method for the solution of a quadratic equation. The chapter ends with Euclid's algorithm and the simplification of fractions with examples.

Euler published some papers where he applied continued fractions to the solution of Riccatti' s differential equation and to the calculation of integrals. He also showed that certain continued fractions derived from power series can converge outside the domain of convergence of the series. In a letter dated 1743 and in a paper published in 1762, Euler investigated the problem of finding the integers a for which a^2+1 is divisible by a given prime of the form $4n + 1 = p^2 + q^2$. Its solution involves the penultimate convergence of the continued fraction for p/q.

In 1765, Euler studied the Pellian equation $x^2 = Dy^2 + 1$ He developed $\sqrt{D}$ into a continued fraction. In 1771, Euler applied continued fractions to the approximate determination of the geometric mean of two numbers whose ratio is as $1/x$. The method can be used to get approximate values of $x^{p/q}$. In 1773, Euler used continued fractions to find x and y making $mx^2 - ny^2$ minimum, and in 1780 for seeking $f$and g such that $fr^2 - gs^2 = x$ In 1783, Euler proved that the value of the continued fraction $\frac{m+1|}{|2} + \frac{m+2|}{|3} + \cdots$ is a rational number, when m is an integer not smaller than 2.

Thus, Euler was the first mathematician not only to give a clear exposition of continued fractions but also to use them extensively to solve various problems. He was quite familiar to the process of transforming a power series into a continued fraction. His method for performing this transformation is simply the division process which is quite similar to Euclid's algorithm for obtaining the g. c. d. of two positive integers. He used this technique in, at least, two papers (: one in 1775 and another in 1776), and he was led to use rational approximations to power series which are, in fact, Padé approximants. In a letter to Christian Goldbach (1690-1764), dated October 17, 1730, Euler considers the series

$$S(x) = x + \frac{1}{2\cdot 3}\frac{x^3}{b^2} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{b^4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 5}\frac{x^7}{b^6} + \cdots,$$

where b is the diameter of a circle, x is the chord and S the corresponding arc. He gives, without any explanations, the following approximations of $S(x)$

$$\frac{60b^2x - 17x^3}{60b^2 - 27x^2} \text{ and } x + \frac{840b^2x^3 - 122x^5}{120b^2(42b^2 - 25x^2)}.$$

It is easy to check that the first approximation satisfies$S(x) + O(x^7)$ and thus is identified with the Padé approximant $[3/2]_S(x)$. The second approximation satisfies $S(x) + O(x^9)$ and thus is identified with $[5/2]_S(x)$ Another important source about Euler's work on rational approximation is its correspondence with the German astronomer Tobias Mayer (1723-1762). On July 27, 1751, Euler answered to Mayer's question on the solvability of the differential equation $dy = (1/logx)dx$ by showing that the series

$$y(x) = -Ux[1 - 1\times U + 1\times 2\times U^2 - 1\times 2\times 3\times U^3 + \cdots + (-1)^\nu \nu!\, U^\nu + \cdots]$$

(with $logx = U$) satisfies this equation. In order to determine the values of this series, Euler wrote that the series

$$S(U) = 1 - 1\times U + 1\times 2\times U^2 - 1\times 2\times 3\times U^3 + \cdots + (-1)^\nu \nu!\, U^\nu + \cdots$$

is equal to the following continuous fraction

$$S(U) = \cfrac{1}{1+\cfrac{U}{1+\cfrac{U}{1+\cfrac{2U}{1+\cfrac{2U}{1+\cfrac{3U}{1+\cfrac{3U}{1+\ddots}}}}}}}.$$

This fraction always closely determines the limits of$S(U)$'s value, and thus one can approximate to the value of $S(U)$ as closely as one will. Then the values approximating to $S(U)$ are:

$$1, \frac{1}{1+U}, \frac{1+U}{1+2U}, \frac{1+3U}{1+4U+2U^2}, \frac{1+5U+2U^2}{1+6U+6U^2}, \frac{1+8U+11U^2}{1+9U+18U^2+6U^3}, \frac{1+11U+26U^2+6U^3}{1+12U+36U^2+24U^3}, \ldots$$

of which every alternate one is greater than S. It is easy to check that the rational fractions given by Euler are the Padé approximants $[0/0]_S(U)$, $[0/1]_S(U)$, $[1/1]_S(U)$, $[1/2]_S(U)$, $[2/2]_S(U)$,... etc.

It must be noticed that Padé approximants can also be found in a letter, dated September 16/27, 1740, of an unknown English mathematician Georges Anderson to William Jones (1675-1749), where Anderson considered Padé approximants to $log(1 + x)$ and he went one step farther that Euler since he gave the first term of the error. About the same time, Daniel Bernoulli (1700-1782) used similar rational fractions in order to invert the power series

$$y = x + ax^2 + bx^3 + \cdots$$

He wants to express x in terms of y. He first writes x as a power series in y. The method of indeterminate coefficients gives

$$x = y - ay^2 + (2a^2 - by)y^3 + \cdots$$

On the other hand, one has

$$1 = \frac{1}{y}x + \frac{a}{y}x^2 + \frac{b}{y}x^3 + \cdots$$

By using his famous method of finding the smallest zero of an infinite power series (published in two memoirs in 1730) applied to difference equations of infinite order, he obtains the sequence

$$\ldots, 0, 0, 1, \frac{1}{y}, \frac{1}{y^2} + \frac{a}{y}, \frac{1}{y^3} + \frac{2a}{y^2} + \frac{b}{y}, \ldots$$

The ratio of two consecutive terms of this sequence gives an approximate value for x. For example, he has

$$x = \frac{y - 3ay^2 + (a^2+2b)y^3 + cy^4}{1 + 4ay + 3(a^2+b)y^2 + (2ab+c)y^3 + dy^4}.$$

Thus, x is approximated by a rational fraction in y. If this rational fraction is developed into an ascending power series in y (by effecting the division), it matches the series obtained from the indeterminate coefficients method up to the term whose degree equals the degree of the numerator. This kind of approximation is weaker than Padé approximation whose degree of approximation is equal to the sum of the degrees of the numerator and the denominator of the rational fraction. Such approximations are now called Padé-type approximations.

However, neither Euler nor Anderson and D. Bernoulli and Johann Heinrich Lambert (1728-1777) (who also gave a direct approach to Padé approximants in his paper "*Observationes variae in Mathesin puram*", published in 1758 in Acta Helvetica) can be credited with the discovery of Padé approximants (or of Padé-type approximants), since they were not aware of their fundamental property of matching the series up to the term whose degree is equal to the sum of the degrees of the numerator and of the denominator (or respectively, of their fundamental property of matching the series up to the term whose degree equals the degree of the numerator).

The first mathematician to be conscious of this property was Joseph Louis Lagrange (1736-1813) in a paper dated 1776 and dealing with the solution of differential equations by means of continued fractions. Transforming the convergents of these continued fractions into rational fractions by using their recurrence relationship he claims that they match the series "*up to the power of x inclusively which is the sum of the highest powers of x in the numerator and in the denominator*". As it is noticed by Brezinski, this paper is really the birth-certificate of Padé approximants.

Many other important contributions to the theory of continued fractions are due to Lagrange. In 1766, he gave the first proof that $x^2 = Dy^2 + 1$ has integral solutions with $y \neq 0$, if D is a given positive integer not a square. The proof makes use of the continued fraction for $\sqrt{D}$. In 1767, Lagrange published a "*Mémoire sur la résolution des équations numériques*", where he gave a method for approximating the real roots of an equation by continued fractions. One year later he wrote an "*Addition*" to the preceding "*Mémoire*", where he proved the converse of Euler's result. He showed that the continued fraction for $\sqrt{D}$ is periodic and that the period can only take two different forms which he exhibited. He related his results to the solution of $x^2 = Dy^2 \pm 1$. In the same paper he extended Huygens' method for solving $py - qx = r$. An interesting problem treated by Lagrange in 1772 is the solution of linear difference equations with constant coefficients. In 1774, in an addition to Euler's Algebra, Lagrange proved that if a is a given positive real number then relatively prime integers p and q can be found such that $p - qa < r - sa$ for $r < p$ and $s < q$ by taking p/q as any convergent of the continued fraction for a in which all the terms are positive. He also gave a method, using continued fractions, to solve $Ay^2 - 2Byz + Cz^2 = 1$ in integers and he proved that Pell's equation cannot be solved by use of a continued fraction for $\sqrt{D}$ in which the signs of the partial denominators are arbitrarily chosen.

Following these predecessors many mathematicians of the nineteenth century became interested by continued fractions. All those who worked on the transformation of a formal power series into a continued fraction, by using for example the division process, have in fact obtained Padé approximants since, in most of the cases, the division process leads to the continued fraction corresponding to the power series whose successive convergents are

$$[0/0], [0/1], [1/1], [1/2], [2/2], \ldots$$

As the history of Padé approximants is very much interlaced with that of continued fractions we shall not follow that way and we shall only look now at the direct approaches to Padé approximants that do not make use of continued fractions. However we would like to mention one more contribution of that type since it opened a very important new chapter in mathematics. In his very celebrated paper on Gaussian quadrature methods, presented to the Göttingen Society on September 16, 1814, Carl Friedrich Gauss (1777-1855) proved that

$$log\frac{1+x}{1-x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots = \frac{1}{x^{-1} + \frac{1/3}{x^{-1} + \frac{2\times(2/3)\times 5}{x^{-1} + \frac{3\times(3/5)\times 7}{x^{-1} + \ddots}}}}.$$

The convergents of this continued fraction are the Padé approximants of the series. The denominators of the convergents are the Legendre orthogonal polynomials as proved by Pafnouty Lvovitch Tchebycheff (1821-1894).

In a paper published in 1837, but dating from November 1834, Ernst Eduard Kummer (1810-1893) made use of Padé approximants for summing slowly convergent series. Kummer writes exactly the equations defining the $[n/n+1]$ Padé approximant, he gives several examples, but he does not prove any theoretical result.

In 1845, Carl Gustav Jacobi (1804-1851) proved his celebrated formula for Padé approximants. In the same paper, he gives several representations for the numerators and the denominators of Padé approximants, all derived as special cases of interpolating rational fractions studied by Augustin Louis Cauchy (1789-1857). Jacobi's representations are based on the systems of linear equations defining the Padé approximants.

Georg Friedrich Bernhard Riemann (1826-1866) proved in October 1863 the convergence of the corresponding continued fraction given by Gauss for the ratio of two hypergeometric series. The proof was found in Riemann's papers after his death. It uses integration in the complex domain, and it was completed and edited by Hermann Amandus Schwarz (1843-1921). According to Henri Eugène Padé (1863-1953), this is the first proof of convergence for Padé approximants.

In his thesis dated 1870, Georg Ferdinand Frobenius (1849-1917) showed that the numerators, the denominators and the errors of the convergents of the continued fraction

$$C(x) = \frac{1}{a_0 x - \frac{1}{a_1 x - \frac{1}{a_2 x - \ddots}}}$$

are related by three terms recurrence relationships. These results were extended in a paper published in 1881 where he gave the relations linking the numerators and the denominators of three adjacent approximants in the Padé table. Some of these identities, now known as the Frobenius identities, are connected with Jacobi's determinant formulas for the coefficients of the continued fraction

$$a_0 + \frac{x}{a_1 + \frac{x}{a_2 + \frac{x}{a_3 + \ddots}}}.$$

The successive convergents of this fraction form the main diagonal of the Padé table. A recursive method for computing a_0,a_1, a_2,... is given by Frobenius who, in fact, gave the first systematic study of Padé approximants and placed their theory on a rigorous basis.

Numerous contributions to Padé approximants are also due to Edmond Nicolas Laguerre (1843-1886). In his first paper of 1876, he treats the cases $(x^2-1)^{-1/2}$, $(x+a)^m/(x+b)^m$and $e^{p(x)}$ where $p(x)$ is a polynomial. In his second note of 1876, he studies $exp(arctanx^{-1})$ and in 1879, he works out the case of the series

$$\frac{1}{x}-\frac{1!}{x^2}+\frac{2!}{x^3}-\frac{3!}{x^4}+\cdots$$

He shows the convergence of the sequence $([k/k]: k \in N_0)$ to $e^x \int_x^\infty e^{-t}t^{-1}dt$, and he also treats the case $\int_x^\infty e^{-t^2}dt$.

In 1881, Leopold Krönecker (1823-1891) considered the problem of finding a rational fraction$p(x)/q(x)$ having the same derivative at a given point that a given function $f(x)$. He used two techniques for solving this problem. The first one is the Euclidean division algorithm for finding the continued fraction expansion of$g(x)/f(x)$. The second method is to solve the system of linear equations obtained by imposing that the first coefficients of the power series expansion of $fq-gp$vanish.

At the same year (1881), in his Inaugural Diploma These, Karl Heun presented the connection between orthogonal polynomials, continued fractions and Padé approximants. Let $(p_\nu: \nu \in N_0)$ be a family of orthogonal polynomials on the closed interval $[\alpha,\beta]$ with respect to a measure $d\mu$that is $\int_\alpha^\beta p_\nu(x)p_\kappa(x)d\mu(x)=0$ (if $\nu \neq \kappa$). These polynomials satisfy a tree terms recurrence relationship $p_\nu(x)=(A_\nu x+B_\nu)p_{\nu-1}(x)-C_\nu p_{\nu-2}(x)$, $\nu \geq 0$. Let us consider the continued fraction

$$\frac{1}{A_1x+B_1-\frac{C_2}{A_2x+B_2-\frac{C_3}{A_3x+B_3-\frac{C_4}{A_4x+B_4-\ddots}}}}.$$

The convergents $R_\nu(x)/S_\nu(x)$ of this fraction are the Padé approximants [0/1], [1/2],....Then

$$S_\nu(x)=\sqrt{C_0}p_\nu(x), \text{ where } C_0=\int_\alpha^\beta x^k d\mu(x).$$

It has been proved in 1896 by Andrei Andrevitch Markov (1856-1922) that if x is an arbitrary point in the complex plane cut along$[\alpha/\beta]$, then

$$\lim_{\nu\to\infty}\frac{R_\nu(x)}{S_\nu(x)}=\frac{1}{C_0^2}\sqrt{C_0C_2-C_1^2}\int_\alpha^\beta\frac{d\mu(x)}{x-t},$$

and that the convergence is uniform on every compact set in the complex plane having no point in common with $[\alpha,\beta]$. Markov's result is a consequence of *Stieltjes' s Theorem* on the convergence of Gaussian quadrature methods.

Another important contribution I would like to mention is that of Charles Hermite (1822-1901). The first reason for that choice is that he was Padé' s advisor, the second reason is that he defined the approximants which are now called the Padé -Hermite approximants. The third reason is that he proved the fundamental result that the number e is a transcendental number and that the proof used Padé approximants. Hermite' s *Proof* is a follows. e is assumed to be an algebraic number, that is satisfying $a_0+a_1e+\cdots+a_ne^n=0$ for some integers $a_0,a_1,\ldots,a_n$ Hermite looks for the polynomials $Q(x)$, $P_0(x)$,...,$P_n(x)$, of degree k, such that $e^{jx}Q(x)-P_j(x)=O\left(x^{(n+1)k+1}\right)$ for $j=0,1,\ldots,n$. Then

$$T(x)=\sum_{j=0}^n a_jP_j(x)-Q(x)\sum_{j=0}^n a_je^{jx}=O\left(x^{(n+1)k+1}\right).$$

Since $|T(1)|<1$ and is an integer for k sufficiently large, it follows that

$$T(1)=\sum_{j=0}^n a_jP_j(1)=0.$$

Giving to k the values k, $k+1$, ..., $k+n$, Hermite proves that $a_0 + a_1 e + \cdots + a_n e^n \neq 0$ which contradicts the assumption. This last part of the proof was quite long and difficult and, in a letter to C.A. Borchardt, Hermite declines to enter on a similar research for the number π. This last step was to be passed by Carl Louis Ferdinand von Lindemann (1852-1939), who, in 1882, proved that π is a transcendental number thus ending by a negative answer a question opened for more than 2000 years! The idea of the proof, which uses Padé approximants, is as follows. If r,s,t,...,z are distinct real or complex algebraic numbers and if a,b,c, ..., n, are real or complex algebraic numbers, at least one of which differing from zero, then$ae^r + be^z + ce^t + \cdots + ne^z \neq 0$. But, $e^{i\pi} + 1 = 0$ and in the preceding result $a = b = 1$ and $c = \cdots = n = 0$; $s = 0$ is algebraic; $r = i\pi$is the only cause why $e^{i\pi} + 1 = 0$. Since i is algebraic, thus $i\pi$is transcendental and it follows that $\pi$is also transcendental.

In his thesis "*Sur la représentation approchée d' une fonction par des fractions rationelles*", which was presented at the Sorbonne in Paris on June 21, 1892 with the jury: Charles Hermite (Chairman and Advisor), Paul Appell (1855-1930) and Charles Emile Picard (1856-1941), Henri Padé (1863-1953) gave a systematical study of the Padé approximants. He classified them, arranged them in the *Padé table* and investigated the different types of continued fractions whose convergents form a descending staircase or a diagonal in the table. He studied the exponential function in details and showed that its Padé approximants are identical with the rational approximants obtained by Gaston Jean Darboux (1842-1917) in 1876 for the same function. He showed that $[n+k/m]_f(t) = \sum_{j=0}^{k-1} c_j t^j + t^k [n/m]_g(t)$, where $f(t) = c_0 + c_1 t + c_2 t^2 + \cdots$ and $g(t) = c_k + c_{k+1} t + c_{k+2} t^2 + \cdots$, and studied the connection between the two halves of the table. Padé also investigated quite carefully what is now called the block structure of the Padé table.

Using a result given by Jacques Hadamard (1865-1963) in his thesis, Robert Fernand Bernard Viscount de Montessus de Ballore (1870-1937) gave, in 1902, his celebrated result on the convergence of the sequence $\left([n/k]_f : n \in N_0\right)$ where f is a series having k poles and no other singularities in a given circle C. Hadamard's results were extended in 1905 by Paul Dienes (1882-1952). This allowed R. Wilson to investigate in 1927 the behavior of $\left([n/k]_f : n \in N_0\right)$ upon the circle $C$and at the included poles.

In 1903, Edward Burr Van Vleck (1863-1943) undertook to extend Stieltjes' theory to continued fractions

$$\cfrac{1}{x+b_1-\cfrac{a_1}{x+b_2-\cfrac{a_2}{x+b_3-\cfrac{a_3}{x+b_4-\ddots}}}},$$

where the a_k's are arbitrary positive numbers and the b_k's are arbitrary real numbers. He connected these continued fractions with Stieltjes' type definite integrals with the range of integration taken over the entire real axis. He also extended Stieltjes' theory to Padé table. The name Padé table has been used for the first time by Van Vleck. In 1914, Hilbert's theory of infinite matrices and bounded quadratic forms in infinitely many variables was used by Ernst Hellinger (1883-1950) and Otto Toeplitz (1881-1940) to connect integrals of the form

$$\int_a^b \frac{d\mu(t)}{x-t} \; (-\infty < a < b < +\infty)$$

with the continued fractions considered by Van Vleck. The same year J. Grommer extended these results to more general cases where the range of integration is the entire real axis. The complete theory was obtained by Hellinger in 1922 using Hilbert's theory of infinite linear systems. The same goal was reached by several other mathematicians (Rolf Hermann Nevanlinna, Torsten Carleman and Marcel Riesz) at about the same time by different methods.

Using the results by Van Vleck, Hubert Stanley Wall (1902-1971), in his thesis dated 1927 under Van Vleck's direction, gave a complete analysis of the convergence behavior of the forward diagonal sequences of the Padé table derived from a Stieltjes series, i.e. whose coefficients are given by

$$c_j = \int_0^\infty t^j d\mu(t),$$

with μbounded and non-decreasing in $[0,+\infty]$. In 1931 and 1932 he extended these results to the cases where the range of integration is $[a,b]$ with $-\infty \le a < b < +\infty$ or with $-\infty < a < b < +\infty$.

The researches on rational approximations during the second part of the twentieth century are mostly devoted to their connection with the theory of orthogonal polynomials and convergence acceleration methods. Since 1965, a growing interest for Padé approximants appeared in theoretical physics, chemistry, electronics, numerical analysis, ... Several international conferences were organized (for example DE BRUIN, M.G. and VAN ROSSUM, H.: *Padé approximation and its applications*. Amsterdam 1980, Lectures Notes in Mathematics 888, Springer Verlag, Heidelberg, 1981; SAFF, E.B. and VARGA, R.S.: *Padé and rational approximation*, Academic Press, New-York, 1977; WUYTACK, L.: *Padé approximation and its applications*, Lectures Notes in Mathematics 765, Springer Verlag, Heidelberg, 1979) and several books were written (for example: BAKER, G.A.jr.: *Essentials of Padé approximants*, Academic Press, New York, 1975; BAKER, G.A. jr. and GRAVES-MORRIS, P.R.: *Padé aprpoximants*, Vols 1 and 2, Encyclopedia of Mathematics and its Applications, Vols. 13 and 14, Addison Wesley, Reading, Mass., 1981; BREZINSKI, C.: *Padé-type approximation and general orthogonal polynomials*, ISNM, Vol. 50, Birskhäuser Verglag, Basel, 1980).

Surely, one the most fundamental and inspired contemporary programs about rational approximation has been that initiated by Claude Brezinski, who was able to extend the notion of Padé approximation by inventing the general theory of Padé-type approximants. Let us understand Brezinski' s motivation. Let f be a formal power series. Padé approximants are rational functions whose expansion in ascending powers of the variable coincides with f as far as possible, that is, up to the sum of the degrees of the numerator and denominator. The numerator and the denominator of a Padé approximant are completely determined by this condition and thus, no freedom is left. If some poles of $f$are known, it can be interesting to use this information. Padé-type approximants are rational functions with an arbitrary denominator, whose numerator is determined by the condition that the expansion of the Padé-type approximant matches the series $f$as far as possible, that is, up to the degree of the numerator. It is also possible to choose some of the zeros of the denominator of the Padé-type approximants (instead of all) and then determine the others and the numerator in order to match the series f as far as possible. Such approximants, intermediate between Padé and Padé-type approximants, have been called higher order Padé-type approximants. Padé approximants are a particular case of Padé-type approximants. The great advantage of Padé-type approximants over Padé approximants lies in the free choice of the poles which may lead to a better approximation.

Much of my scientific work is devoted to the development of the theory of Padé-type approximants ([1],[2], [6], [8], [9], [12], [13], [15], [16], [17], [18], [22], [25] and [29]).

- History of deterministicmathematical combat theories

The paper [23] presents an exhaustive and comprehensive overview of the published works in Mathematics, that contributed to the foundation of deterministic combat models.

Stochastic Modeling and Numerical Simulation in Operations Research

My research work on Stochastic Modeling and Numerical Simulation in Operations Research is directed towards the three(3) main specializations: *Stochastic modeling in military operations research (Stochastic and renewal combat models, missile allocation strategies, reliability of military operations, target coverage, optimal placement of defense forces, strategic defense) and Numerical Multi-Agent Simulation (in small to medium scale).*

- ## Stochastic modeling in military operations research

  - ### Stochastic and renewal combat models

    The next two specializations of my work on the stochastic modeling in military operations research focus on *stochastic combat models* and *renewal combat models* ([20], [21], [82], [83], [84] and [85]).

    Throughout recorded history, military historians, philosophers, and generals have proposed various principles of war. These principles are stated as precepts with victory as the goal. It is important to recognize that these precepts and their implications are qualitative and not quantitative in nature.

    At the beginning of the 20th century (ca 1914), F.W. Lanchester, in England, developed mathematical expressions for the connections between losses and force sizes of opposing military forces[22]. At roughly the same time (ca 1915), M. Osipov, in Russia, developed the same concepts[23]. Both were preceded by J.V. Chase (1902), in the U.S.A.[24]. For historical reasons, we use the terms "*Lanchester laws*" or "*Lanchestrian attrition*" to refer to these mathematical formulas.The Lanchester laws are given by a set of coupled ordinary differential equations as models of attrition in modern warfare. For the simplest case of directed fire, for example, they embody the intuitive idea that one side's attrition rate is proportional to the opposing side's size[25]. With the advent of computers, simulations of warfare have been possible in excruciating detail. However, the production of probable outcomes rests on unvalidated models of combat activities: a validated (Lanchestrian or not) attrition algorithm does not exist because of the data great problem [26]. On the other hand, Lanchecter laws are applicable only under a strict set of assumptions, such as having homogeneous forces that are continually engaged in combat, firing rates that are independent of opposing force levels are constant in time, and units that are always aware of the position and condition of all opposing units. Lanchester laws also contain a number of significant shortcoming, including modeling combat as a deterministic process, requiring knowledge of "*attrition rate coefficients*" (the values of which are, in practice, very difficult to obtain), inability to account for any sup-pressive effects of weapons, and failure to account for terrain effects. Conceptually, there are two significant drawbacks to using Lanchester laws to model land combat. First, the Lanchester laws are unable to account for any spatial variation of Forces (no

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22 See F.W. Lanchester, *Aircraft in Warfare*, *Engineering*, Vol. 98 (1914), pp. 422–423; reprinted on pp. 2138–2148, in *The World of Mathematics*, Vol. IV, J. Newman, Simon and Schuster (editors) (1956).

23 See M. Osipov, *The influence of the numerical strength of engaged forces in their casualties*, translated by R.L. Helmbold and A.S. Rehm, *Naval Research Logistics*, Vol. 42 (3) (April 1995), pp. 435–490.

24 J.V. Chase, A Mathematics Investigation of the Effect of Superiority in Combats Upon the Sea, 1902; reprinted in B.A. Fiske, The Navy as a Fighting Machine, Annapolis, MD, U.S. Naval Institute Press, 1988.

25 We refer readers who are not familiar with Lanchestrian Attrition to J. G. Taylor, Lanchester Models of Warfare, 2 Volumes, Military Applications Section of the Operations Research Society of America, 1983 and D.S. Hartley, III, Topics in Operations Research: Predicting Combat Effects, Military Applications Society, INFORMS, 2001.

26 See Chapter 2 of D.S. Hartley, III, Topics in Operations Research: Predicting Combat Effects, Military Applications Society, INFORMS, 2001

link is established, for example, between movement and attrition). Second, they do not incorporate the human factor in combat (i.e., the psychological and/or decision-making capability of the human combatant). For the first drawback confronting, many simulations of warfare use a Lanchestrian attrition algorithm as the driver for other algorithms, such as advance rates and deciding victory. The second drawback remains an insuperable obstacle. There have been many extensions to and generalizations of the Lanchester laws over the years, all designed to minimize the deficiencies inherent in their formulation, mainly including reformulations as stochastic differential equations and partial differential equations[27].

In recent years, deterministic and stochastic Lanchester laws are both disputed. So, in 1996-2004, A. Ilachnski was able to describe land combats as complex adaptive systems [28]. Another direction, independent of Ilachinski's ideas, consists in a pure stochastic description of combat's evolution.

The paper *«Non Lanchestrian stochastic processes in War Analysis. Part I»* ([20]) deals with an introduction to the theory of war's stochastic processes[29]. First, we define the stochastic process of losses in a combat and show that

> Theorem 22 ([20]). *The distribution function for a losses' process is a Poisson distribution with a parameter equal to combat's intensity.*

Next, we consider intermediate times T_v between successive losses, and we prove that

> Theorem 23 ([20]). *The set* $\{W_v : v = 1,2,\dots\}$ *where* W_v *is the waiting time up to the* v^{th} *loss forms a stochastic process with probabilities given by an Erlang distribution.*

Moreover, it is showed that

> Theorem 24 ([20]). *The successive intermediate times* $T_1, T_2, T_3, \dots$ *between successive losses are independent and equidistributed random variables, with common exponential density function.*

The randomness character for a stochastic process of losses is also stated.

> Theorem 25 ([20]). *If* $\{A(t) : t \geq 0\}$ *is the stochastic process of losses for a fighting military force in a combat with intensity* λ*, then the conditional distribution for the moments* $t_1 < t_2 < \cdots < t_v$ *of occurrence of the first v successive losses (assuming that* $A(t) = v$*), is exactly the same as the distribution of an ordered random sample of v uniformly distributed observations on time interval* $[0, t]$*. In other words, the corresponding non ordered moments* $t'_1, t'_2, \dots, t'_v$ *of occurrence of v losses (given that* $A(t) = v$*), constitute a uniformly distributed random sample on time interval* $[0, t]$*.*

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27 A comprehensive review and critique of Stochastic Lanchester Laws is given in: C.J. Ancker, Jr. and A.V. Gafarian, Modern Combat Models, a Critique of Their Foundations, Topics in Operations Research, ORSA, 1992.

28 See A. Ilachinski, Artificial War: Multiagent-Based Similation of Combat, World Scientific Publishing, Singapore, 2004.

29 For a lucid study of stochastic processes (: connected stochastic series of events), one can consult W. Feller, An Introduction to Probability Theory and its Applications,Vol. II, John Wiley, New York, 1971; S. Karlin and H. M. Taylor, A First Course in Stochastic Processes, 2nd edition, Academic Press, 1975; V. K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, John Wiley and Sons, 1976; and D. Stirzaker, Stochastic Process and Models, Oxford University Press, 2005.

Theorem 26 ([20]). *If* $\{A(t): t \geq 0\}$ *is the stochastic process of losses for a fighting military force, then for every* $0 < s \leq t$ *and* $k \leq v$*, the conditional distribution of* $A(s)$*, assuming that* $A(t) = v$*, is the binomial distribution*

$$b\left(\frac{k}{v}, p\right)$$

with $p = s/t$*. In other words, whenever* $0 < s \leq t$ *and* $k \leq v$*, it holds*

$$P(A(s) = k/A(t) = v) = \binom{v}{k}\left(\frac{s}{t}\right)^k\left(1 - \frac{s}{t}\right)^{v-k}.$$

Some reasonable and natural generalizations are also given in [20]. Further, we define and study stochastic processes of losses and reinforcements with *difficult access* in the combat. After investigating the probability distribution of the fighting size $X(t)$ of a military force, we get combat's Chapman-Kolmogorov equations, and subsequently combat's Kolmogorov forward stochastic differential equations on corresponding transition probabilities.

The above theory generalizes to linear stochastic processes of losses and reinforcements in case where only a portion of reinforcing units has difficult access in the combat ([20]). We also include a report on stochastic differential equations ruling non-homogeneous processes of losses and reinforcements with difficult access in the combat. As an application, we compute *the destruction probability for all units of a military force at any moment of the combat*, and *at the end of the combat*. Finally, we outline combat's Kolmogorov differential equations in the general case of a non-homogeneous stochastic process of losses and reinforcements with difficult access in the combat.

Our aim in the paper *«Non Lanchestrian stochastic processes in War Analysis. Part II: the renewal process of combat's losses»* ([21]) is to give an introduction to the renewal process of combat's losses. The conception of the renewal process is woven together with the conception of the stochastic sequence of events that are distributed randomly into the time interval $]0,1[$. An example of renewal process is the (homogeneous or not homogeneous) stochastic process of losses in a combat ([21]). Remind that a random variable $A(t)$ representing the total number of fighting military force's losses on account of opponent fires up to the time moment t from the beginning of a combat is called a non homogeneous stochastic process of losses, if

(i). 1st Property: The number $A(0)$ of losses at the beginning of the combat is equal to zero,

(ii). 2nd Property: For any $h > 0$, the differences

$$A(t) - A(s) \text{ and } A(t+h) - A(s+h)$$

are equidistributed events, in the sense that for any $\kappa = 0,1, \ldots$ it holds

$$P(A(t+s) - A(s) = \kappa) = a_\kappa(t) \text{ whenever } t \geq 0 \text{ and } s \geq 0$$

(: we say that *the losses of the fighting military force have stationary increases*),

(iii). 3rd Property: For any $v > 2$ and any $t_1 < t_2 < \cdots < t_v$ the differences

$$A(t_2) - A(t_1), A(t_3) - A(t_2), \ldots, A(t_v) - A(t_{v-1})$$

are independent events (: we say that *the losses of the fighting military force have independent increases*),

(iv). 4th Property: There is at most one loss occurring into any infinitesimal time interval $]t, t+h[$; in other words, there is a function $\lambda(t)$ such that

$$a_1(h) = P(A(t+h) - A(t) = 1) = \lambda(t)h + o(h)$$
$$a_0(h) = P(A(t+h) - A(t) = 0) = 1 - \lambda(t)h + o(h);$$

where the "remainder" $o(h)$ is sufficiently small, in the sense that

$$lim_{h\to 0}\frac{o(h)}{h} = 0.$$

The function $\lambda(t)$ in *Property (iv)* above is the *intensity* of the stochastic process of combat's losses. Now, the function $A(t)$, $t \geq 0$, is called the *renewal counting function of combat's losses*. In case where the intensity is a constant parameter, the stochastic process $A(t)$ is said to be a *homogeneous stochastic process of combat's losses*. For such a (homogeneous) process, the intermediate times between successive losses are independent and equidistributed random variables, with common exponential distribution density (see theorem 2.4 in [33]).

A natural and direct generalization is obtained by considering losses with independent and equidistributed intermediate successive times, but without exponential (common) distribution density. In [21], we discuss this generalization, by studying the renewal process of combat's losses, that is the set $(T_v: v = 1,2,\dots)$ of intermediate times between successive combat's losses.

Theorem 27 ([21]). *Let* $(T_v: v = 1,2,\dots)$ *be a renewal process of combat's losses with (common) general probability density function* $f(x)$ *and (common) general probability distribution* $F(t)$.

i. *For any piecewise bounded (: bounded on every bounded time interval) function* $g(t)$, *there is one and only one piecewise bounded function* $G(t)$ *satisfying the following renewal equation of losses for this combat:*

$$G(t) = g(t) + \int_0^t G(t-x)dF(x),\ t \geq 0.$$

ii. *The exact form of* G *is given by the formula*

$$G(t) = g(t) + \int_0^t G(t-x)dM(x),\ t \geq 0$$

where $M(t)$ *is the losses' renewal counting function that corresponds to the above probability distribution of combat's losses, that is the function* $M(t)$ *satisfying the following integral equation*

$$M(t) = F(t) + F * M(t) = F(t) + \int_0^t M(t-x)dF(x),\ t \geq 0.$$

As a first application, we prove a result for the *renewal counting function of combat's losses* $A(t)$, $t \geq 0$ and we give the basic property on the expected number $M(t) = E\big(A(t)\big)$ of combat's losses in time interval $[0,t]$.

Theorem 28 ([22]). *The expected value for the time interval length required to express the number of losses up to the final moment of this interval as the sum of the number* $A(t)$ *of losses up to the moment* t *plus one more loss is equal to the product of the expected value for the waiting time up to the first loss and the expected value for the sum of losses's number* $A(t)$ *up to the moment t plus one more loss:*

$$E\big(S_{A(t)+1}\big) = E(T_1)E(A(t)+1).$$

(Here, the function S_v is defined to be $S_v = T_1 + T_2 + \cdots + T_v$.)

ii. *The expected value for the time interval length required to* *express the number of losses up to the final moment of this interval as the sum of the number* t) *of losses up to the moment* t *plus one more loss is equal to the product of the expected value for the waiting time up to the first loss and the sum of losses' expected value in the time interval* $]0, t[$*plus one more loss:*

$$E(S_{A(t)+1}) = E(T_1)[M(t) + 1].$$

Next,we prove renewal theorems on combat's losses.

Theorem 29 ([22]). *Let* $(T_v : v = 1,2, \dots)$*be a renewal process of combat's losses with expected value* $\mu = E(T_1) < \infty$ *for the waiting time* T_1*up to the first loss. Then, as combat's duration grows, the losses' mean per unit of time approaches the quantity* $1/\mu$*:*

$$lim_{t\to\infty} \frac{M(t)}{t} = \frac{1}{\mu}.$$

Theorem 30 ([22]). *As combat's duration grows, the losses' mean into an interval time of length* h*approaches the quantity* h/μ*, where* $\mu = E(T_1)$ *is the expected value for the waiting time up to the first loss:*

$$M(t + h) - M(t) \underset{t\to\infty}{\longrightarrow} \frac{h}{\mu} \text{ whenever } h > 0.$$

Further, we describe exact and asymptotic probability distributions for the three main loss' time moments (: the following loss' time moment, the preceding loss' time moment and the successive losses' time moments).

Theorem 31 ([22]). *Let* $F(t) = P(T_v \leq t)$ *be the probability distribution function for a renewal process* $(T_v : v = 1,2, \dots)$ *of combat's losses, with mean* $\mu = E(T_1) \neq 0$ *and renewal counting function of losses* $M(t) = E(A(t))$*.*

i. *For a probabilistic description of the random variable* $\gamma(t) := S_{A(t)+1} - t, t > 0$*, which represents the function of time for the following loss in a combat, it is useful to consider the function* $G_T(t) := P(\gamma(t) > T)$*,* $T > 0$*. The function* $G_T(t)$ *is called the exact probability distribution for time moments of the following losses. The exact probability distribution for time moments of the following losses is the function*

$$G_T(t) = 1 - F(t + T) + \int_0^t [1 - F(t + T - x)]dM(x), \quad T > 0.$$

ii. The limit $G_T = lim_{t\to\infty} G_T(t)$ (if it exists) is called *the asymptotic probability distribution for time moments of the following losses during a time interval of length* T*. If* $\mu = E(T_1) \neq 0$*, then the asymptotic probability for time moments of the following losses during a time interval of length* T*exists and is equal to*

$$G_T = \frac{1}{\mu} \int_0^\infty [1 - F(y)]dy.$$

Theorem 32 ([22]). *If* $F(t) = P(T_v \leq t)$*be the probability distribution function for a renewal process* $(T_v : v = 1,2, \dots)$ *of combat's losses, with mean* $\mu = E(T_1) \neq 0$*. The random variable* $\delta(t) := t - S_{A(t)+1}, t > 0$ *represents the time from the preceding loss in a combat. Define* $H_T(t) := P(\delta(t) > T)$*,* $T > 0$*.The function* $H_T(t)$ *is called the exact* *probability distribution of time moments from the preceding losses. The asymptotic probability distribution for the time moments of the preceding losses during a time interval of length* T *is the limit*

$$H_T = lim_{t\to\infty} H_T(t) \text{ (if it exists).}$$

It holds

$$H_T = \frac{1}{\mu}\int_0^T [1 - F(x)]dx.$$

Theorem 33 ([22]). *If* $F(t) = P(T_v \leq t)$ *be the probability distribution function for a renewal process* $(T_v: v = 1,2, \dots)$ *of combat's losses, with mean* $\mu = E(T_1) \neq 0$*. The random variable*

$$\beta(t) := \gamma(t) + \delta(t) = t - \left(S_{A(t)+1} - S_{A(t)}\right), t > 0$$

represents the duration length of the time interval between two successive combat' losses. Define $K_T(t) := P(\beta(t) > T),\ T > 0$*. The function* $K_T(t)$ *is called the exact probability distribution of duration length between two successivecombat' losses. The asymptotic probability distribution for the time moments of the preceding losses during a time interval of length* T *is the limit* $K_T = lim_{t\to\infty} K_T(t)$ (if it exists). *It holds*

$$K_T = \frac{1}{\mu}\int_0^\infty y dF(y).$$

Moreover, we outline a model for substituting fighting units and we study to the so-called *stationary renewal process of combat's losses*. In this connection, we have the

Theorem 34 ([22]). *Suppose that, for a stationary renewal process of losses in a combat, the probability distribution* $F_1(T)$ *for the waiting time* T_1 *up to the first loss is given by the formulae*

$$F_1(T) = \frac{1}{\mu}\int_0^T [1 - F(t)]dt,$$

where $F(t)$ *is the common probability distribution function of the intermediate times* $T_2, T_3, \dots$ *between first and second loss, second and third loss, etc. Let* $A_s(t)$ *be the associated counting function of losses. The corresponding renewal counting function of combat's losses is*

$$M_s(t) := E\left(A_s(t)\right) = t/\mu,$$

and the corresponding probability distribution for the time moments of the next losses is

$$P(\gamma_s(t) \leq T) = F_1(T), T > 0.$$

Finally, we prove that *the counting function A(t) of Combat's losses has asymptotic normal distribution with asymptotic mean and variance two asymptotic properties for the counting function* $A(t)$ *of combat's losses*. In fact

Theorem 35 ([22]).(The Central Marginal Theorem for the Counting Function of Combat's Losses). *If the probability distribution* $F(t)$ *has mean* $\mu = E(T_2) < \infty$ *and variance* $\sigma^2 = Var(T_2) < \infty$*, then, whenever* $T > 0$*, it holds*

$$lim_{t\to\infty} P\left(\frac{A(t) - \frac{t}{\mu}}{\sqrt{\frac{t\sigma^2}{\mu^3}}}\right) = \Phi(t),$$

where Φ *is the reduced normal distribution function.*

Theorem 36 ([22]). (The Strong Law of Large Numbers for the Counting Function of Combat's Losses). *If the probability distribution* $F(t)$*has mean* $\mu = E(T_2) < \infty$ *and variance* $\sigma^2 = Var(T_2) < \infty$*, then*

i. *it is certain that the value of the ration* $A(t)/t$ *approaches the number* $1/\mu$*as Combat's duration* t*increases:*

$$P\left(lim_{t\to\infty}\frac{A(t)}{t} = \frac{1}{\mu}\right) = 1;$$

ii. *as Combat's duration* t*increases, the value of the ratio*

$$\frac{\Delta[A(t)]}{t} := \frac{A(t+h) - A(t)}{t}$$

approaches the number σ^2/μ^3*:*

$$lim_{t\to\infty}\frac{\Delta[A(t)]}{t} = \frac{\sigma^2}{\mu^3}.$$

Besides the above research papers, the monographs «Operations Research and its Military Applications. Introduction to the Stochastic and Renewal Theories of War. Volume 1. Issue 1» ([82]) and «Operations Research and its Military Applications. Introduction to the Stochastic and Renewal Theories of War. Volume 1. Issue 2» ([83]) collect and explain fundamental methods for an exhaustive and informative first undergraduate course on stochastic and renewal combat models.

If conventional warfare is a complex situation that is difficult to model in the best of circumstances, then unconventional warfare is a nightmare for military analysts. Irregular strategies rely heavily on deception and deceit. Many aspects of this type of Irregular Warfare (IW), such as recruitment, desertion, psychological warfare, etc. are based on individual human behavior and are very difficult to model. Our primary goal in [104] is to provide a descriptive probabilistic method and efficient mathematical tool to help understand cause-and-effect relations in the context of IW directly related to the involved military forces or populations support and control.

The paper begins by providing a short explanation and review of contemporary IW attrition models, including terrorism-counter-terrorism struggle models, epidemiological models, spatial-temporal models, adversarial organizations destabilization models and risk assessment and decision making models.

Despite emerging perspectives, all these methods seem to be applicable only under a strict set of assumptions, such as having homogeneous forces that are continually engaged in combat, firing rates that are independent of opposing force levels are constant in time, and units that are always aware of the position and condition of all opposing units. All these models also contain a number of significant shortcoming, including modeling combat as a deterministic process, requiring knowledge of crucial and mutable "coefficients" (the values of which are, in practice, very difficult to obtain), inability to account for any suppressive effects of weapons, and failure to account for population behavior and/or terrain effects. Conceptually, there are two significant drawbacks to using these methods to model IW. First, these models are unable to account for any spatial variation of opposing forces and population (no link is established, for example, between movement and attrition). Second, they do not incorporate the human factor in combat (i.e., the psychological and/or decision-making capability of the human combatant and the member of population).

To address this deadlock, in [104], we develop an IW's stochastic and renewal process theory for losses and escapes. To do so, we consider the stochastic (random) variable $A(t)$ denoting the total losses number of an agency or population Y involved in an IW's conflict up to the moment time $t$from the beginning of the conflict. The first main result is that the distribution function for the associated IW–losses process with intensity λ is a Poisson distribution with parameterλ.

Next, we consider intermediate timesbetween successive losses for an agency or population Y involved in the IW's conflict, and we prove that the set $(W_\nu : \nu = 1,2, \dots)$, where W_ν is the waiting time up to the ν^{th} IW–loss, forms a stochastic process with probabilities given by an Erlang distribution. Moreover, it is showed that the successive intermediate times $T_1, T_2, T_3, \dots$ between successive losses of an agency or population$Y$ involved in the IW's conflict are independent and equidistributed random variables, with common exponential density function. The randomness character for a stochastic process of IW–losses will be investigated and some reasonable and natural generalizations will be given.

Next, we consider and study stochastic processes of IW–losses and reinforcements for a force with difficult access in the IW's conflict. After studying the fighting size probability distribution of Y's remaining units, we will get IW–Chapman-Kolmogorov equations, and subsequently IW's Kolmogorov forward stochastic differential equations on corresponding transition probabilities. The above theory will be generalize to the linear stochastic process and stochastic differential equations ruling non-homogeneous processes for the IW–losses/reinforcements of any involved force in case where only a portion of reinforcing units has difficult access in the IW's conflict. In the same context, we describe the linear stochastic process and stochastic differential equations ruling non-homogeneous processes for the IW–losses/escapes of any involved population in case where only a portion of population units has limited ability to escape. As an application, we compute the complete destruction probability of all involved force's units, as well as the complete destruction probability of all involved population units at any moment of the IW, specifically at the end of the IW. We also outline the IW's Kolmogorov differential equations for the non-homogeneous stochastic process of IW–losses/reinforcements of a force with difficult access in the IW and, also, the IW–Kolmogorov differential equations of losses/escapes for an IW involved population with limited ability to escape.

A natural and direct generalization is obtained by considering IW– losses with independent and equidistributed intermediate successive times, but without exponential (common) distribution density. We discuss this generalization, by studying the renewal processes of successive IW–losses for each force or each population involved in an IW's conflict. Now, the function $A(t), t \geq 0$ is called the renewal counting function for the IW– losses of each force or population involved in the IW's conflict. We also give basic definitions and properties on the expected number $M(t) = E\big(A(t)\big)$ of IW's conflict losses in any time interval$[0, t]$ and wepresent renewal theorems for the IW's conflict losses. We also describe the exact and asymptotic probability distributions for the three main time moment of IW–losses (: the following loss' time moment, the preceding loss' time moment and the successive losses' time moments). Next, we outline a model for substituting fighting units, and we will describe to the so-called stationary renewal process of the IW's conflict losses. Finally we formulate asymptotic properties for the counting function $A(t)$.

- **Reliability**

  In [107], after defining different types of system operations (such as a series system of operations, a system of parallel operations, a system of operations with active or standby redundancy, a system of combined operations), we study the operational reliability variation as a function of the time. In particular, we consider reliabilities with different types of distributions (exponential distribution, normal distribution, log-normal distribution and Weibull distribution). We also study the instantaneous failure rate of the system operations, the so-called hazard function, and we provide experiment computation of the reliability function using the probability graph method and the Kolmogorov-Smirnov test method. Further, we investigate the maintainability of weapon systems and the operational availability of military systems.

- **Missile allocation strategies**

  In the papers *«Missile allocation strategies for a group of identical targets. Part I»* ([72]), *«Missile allocation strategies for a group of identical targets. Part II»* ([73]), as well as in the monograph «Missile Allocation Strategies» ([92]), we collect and explain several mathematical models for a first missile allocation undergraduate course. Among other, key issues discussed here are s*trategies offense-last-move and defense-last-move strategies for a group of identical targets*, *strategies for a group of identical targets when neither side knows the other's allocation*, *nonpreallocation strategies for a group of identical targets*, *group preferential defense strategies for a group of identical targets*, d*efense damage assessment strategies for a group of identical targets*, *attacker-oriented defense strategies for a group of identical targets*, *offensive damage assessment strategies for a group of identical targets*, *offense allocation to a group of targets with different values in the no-defense case*, *general techniques for one-sided allocation problems for a group of targets with different values*, *general methods for constructing two-sided offense-last-move strategies for a group of targets with different values*, *two-sided offense-last-move strategies for a group of targets with different values using payoff functions*, *two-sided offense-last-move strategies for reliable missiles*, *offense and defense preallocation strategies for a group of targets with different values when neither side knows the other's allocation*, *defense strategies for a group of targets with different values when the offensive stockpile size is unknown*, *attacker-oriented defense strategies for a group of targets with different values*, *attacks on the defense systems* (*models involving reliable missiles and soft radars*, *models involving radars resistant to damage*, *attacks on defensive missile silos*, *attacks on command and control systems*), *mixtures of local and area defense missiles*, *models for local and area missiles involving costs* and *offense and defense strategies for targets of unequal value*.

  Besides the above mentioned documents, the issuedundermy supervision book «Missile Allocation Strategies and Multi-Agent Based Simulation of Combat, Volume 1» ([72]) is the outgrowth of the Conference with the same title held in November 2008 at the Hellenic Military Academy and contains theoretical results relating to missile allocation strategies.

- **Target coverage**

  In the monograph «Target Coverage» ([91]), we arelaying the fundamentalsof a modern attack target theory. Among other, key issues discussed here are *survival/destruction probabilities for one or more point targets*, *expected fractional damage of a uniform-valued circular target*, *expected fractional damage of a Gaussian target*, *the diffused*

Gaussian damage function, attack dispersion to an area target, estimating probability of survival/destruction from impact-point data, offensive shoot-adjust-shoot strategies, attack evaluation by defense using radar information, defense strategies against weapons of unknown lethal radius, defense strategies against a sequential attack of unknown size, defense strategies against a sequential attack by weapons of unknown lethal radius, defense strategies against a sequential attack contain exactly one weapon mixed with decoys, shoot-look-shoot defense strategies.

Besides the above mentioned monograph, the issuedundermy supervisionbook«Missile Allocation Strategies and Multi-Agent Based Simulation of Combat, Volume 1» ([72]) is the outgrowth of the Conference with the same title held in November 2008 at the Hellenic Military Academy and contains theoretical results relating to target analysis and target analysis.

- Optimal placement of defense forces

The fourth specialization of my work on (Military) Operations Research deals with the problem of optimal placement of defense forces. In the paper «*Optimal Placement of Defense Forces*» ([32]) we present and study an effective methodof defenseaimingat minimizing theterritoriesto conquerthe attacking.

Further, in [105], we investigate the numerical solution of the defense force positioning problem in order to handle in an efficient way the forces of the attacker[30]. The scope is the minimization of the conquest territories[31]. The basic types of defense are the distributed static and the concentrated mobile defense. The defense forces should swoop rapidly to any point of the defense locus in order to protect their territories. The selection of the "optimal" defense forces position should be placed is a difficult problem and it aims at the minimization of enemy's penetration. This minimization results to systems of non linear equations. There are many classical methods for solving such systems. The most known one is Newton's method which requires the computation of the inverse of the Jacobian matrix at every step. This can be avoided by solving an equivalent system of equations. Another well known algorithm for solving the previous non linear system of equations is Broyden's method. Many other approaches, such as quasi-Newton methods[32], solve the system in a faster way by approximating the inverse of the Jacobian matrix reducing significant the required complexity of the algorithm[33]. The main difficulty in all the pre-mentioned methods is the selection of the initial point, which has many times to be very close to the solution[34]. We study the behavior of the procedures for various initial points and small perturbations of the data in order to present stable procedures which compute efficiently the solution of the system of non linear equations, leading to the optimal selection of the position, on which the forces of the defender should be placed. All the proposed methods are tested for various sets of data and useful conclusions arise. The algorithms are compared in respect of the computational complexity and stability through error analysis concluding to useful results.

30 See N. J. Daras: Operations Research and its Military Applications. Volume2. Issue2. Theater Missile Defense and Tactical Engagements of Heterogeneous Forces(e-book, Hellenic Military Academy, Vari Attikis, Greece, 2013,pp. 143, http://www.sse.gr/files/Epixeirisiaki_Erevna_kai_Stratiotikes_Efarmoges_Aftis_Tomos2_Vivlio2.pdf (also http://ekeo.gr/wp-content/uploads/reports/daras/operations2.pdf).

31 See R. Gupta, Defense Positioning and Geometry, the Brookings Inst., Washington, DC, 1993.

32 See C. Brezinski, A classification of quasi-Newton methods, Numer Algor, Vol. 33, 123-135, 1997.

33 See C. Brezinski, Projection Methods for Systems of Equations, North- Holland, Amsterdam, 1997.

34 See D. J. Kavadias and M. N. Vrahatis, Locating and computing all the simple roots and extrema of a function, SIAM J. Sci. Comput., Vol. 17, 1232-1248, 1996.

- **Strategic defense**

  The two (2) monographs entitled «Operations Research and its Military Applications. Volume 2. Issue 1: Strategic Defense» ([84]) and «Operations Research and its Military Applications. Volume 2. Issue 2: Theater Missile Defense and Tactical Engagements of Heterogeneous Forces» ([85]) referto implementations of *layered defense*, *antiballistic missile defense*, *optimal penetration routes through air defense*, *theater missile defense*, *tactical engagements of heterogeneous forces*, *aggregated forces* and *superiority parameters*.

  In this direction, the first part of [103] is devoted to the definition and study of a layered defense against simultaneous offensive weapons. Then we discuss the layered defense against offensive weapons depending on the defensive or offensive strategy to be adopted by each opposing side. The solution comes with the help of game theory and is optimal both for defense and for attack. Next, we give a solution to the problem of determining optimal penetration routes through a defensive grid. The resolution process includes the concept of the threat function, as well as the analysis of an effective defense against attacking fighters. The paper concludes with a description of the overall layered defense effectiveness against attacking invaders when the two opponent forces adopt allocated ballistic strategies. This case defines the concept of the theater ballistic defense. Towards this direction, we examine three different tactics against offensive weapons: (1) random assignments of targets, (2) uniform assignments of targets and (3) shoot-look-shoot assignments of targets.

- **Numerical Multi-Agent Simulation**

  The issuedundermy supervision book «Missile Allocation Strategies and Multi-Agent Based Simulation of Combat, Volume2» ([73]) is the outgrowth of the Conference with the same title held in November 2008 at the Hellenic Military Academy and is dedicated to models of artificial war and numerical multi-agent simulation digital programs (in small to medium scale) using autonomous agents to model personal behavior and reactions during a combat depending on environmental conditions. Topics covered include: *Nonlinearity, complexity and warfare, Simulation experimental analysis, Operational effectiveness evaluation based on statistic analysis, Multi-agent simulation as a tool for modeling societies, Calulation methods, Parallel architectures, Computational and dara grids, Combat simulation systems,Network centric models for warfare, Global numeric validation.*

- **Writings**

  The monograph «Probability Theory and Applications in Military Affairs» ([90]) lays thenecessarygroundwork fordeveloping a stochastic theory with application to several military problems.

- **Edition of books and special issues**

  The issuedundermy supervision book «Applications of Mathematics and Informatics in Military Science» ([58]) is an outgrowth of a conference held in April 2011 at the Hellenic Military Academy andbrings together a wide variety of mathematical methods with application to defense and security and discusses directions and pursuits of scientists that pertain to the military. Also studied is the theoretical background required for methods, algorithms, and techniques used in military applications as well as the direction of theoretical results in these applications. Open problems and future areas of focus are also highlighted. Topics covered include: *applied operations research and military applications*, *signal processing*, *scattering*, *scientific computing and applications*, *combat simulation and statistical modeling*, *satellite* *remote sensing*, *applied informatics – cryptography*, *coding*. Among the contents, mention the following specific issues: *Selected topics in critical element detection, Study of engagement with mobile targets, Solving an electromagnetic scattering problem in chiral media, Orthonormality in interpolation schemes for reconstructing signals, Computer graphics techniques in military applications, Numerical optimization for the length problem, Adaptive policies for sequential sampling under incomplete information and a cost constraint, On a Lanchester combat model, land warfare and complexity, Wavelet transform in remote sensing with implementation in edge detection and noise reduction, Optimal orbital coverage of theater operations and targets, A bird's-eye view of modern symmetric cryptography from combinatorial designs, On the weak convergence of an empirical estimator of the discrete-time semi-markov kernel, Analysis methods for unreplicated factorial experiments.*

In this direction, the issued under my supervision book «Applications of Mathematics and Informatics in Science and Engineering» ([59]), as well as the special issues «Mathematics and Informatics in Military Science» ([63], [64]), contain selected scientific contributions presented at a Conference held in April 2013 at the Hellenic Military Academy and bring together a broad variety of mathematical methods and theories with several applications. Topics covered include: *operations research /operational analysis organization, combat models / simulation of operations, systems of administration and control, military logistics chain, humanitarian & relief logistics, portfolio of national defense, game theory, missile allocation strategies and target coverage, scientific computing and applications, ballistics/ modern arming systems, laser radiation / directed energy weapons, geography of modern arming technologies, geography of modern arming technologies, new cryptographic methods, cryptology and computational number theory, cyber war/security, statistical modeling and applications, systems of monitoring and spatial analysis, digital signal processing/ pattern recognition, robotics, automatic control and intelligent systems, satellite communications and remote sensing, modern radar techniques, unmanned vehicles, multiplicative factors of force*.

Among the contents of [59], mention the following specific issues: *Almost periodic solutions of Navier-Stokes-Ohm type equations in Magneto-hydrodynamics, Scattering relations for a multi-layered chiral scatterer in an achiral environment, Stochastic analysis of cyber-attacks, Numerical solution of the defense force optimal positioning problem, Invisibility regions and regular meta-materials, Error bounds for trapezoid type quadrature rules with applications for the mean and variance, Evaluating UAV impact in the tactical context of a mechanized infantry scout platoon through military simulation software, Balanced integer solutions of linear equations, Satellite telecommunications in the military: advantages, Limitations and the networking challenge, Operational planning for military demolitions: an integrated approach, Stabilization and tracking for swarm based UAV missions subject to time-delay, Correlated phenomena in wireless communications: a copula approach, A control scheme towards accurate firing while moving for a mobile robotic weapon system with delayed resonators, Reliability analysis of coherent systems with exchangeable components,Abductive reasoning in 2D geospatial problems, About model complexity of 2-D polynomial discrete systems. An algebraic approach, Computational number theory and cryptography, Improvement of order performance of a supplier, Parametric design and optimization of multi-rotor aerial vehicles, A multidimensional Hilbert-type integral inequality related to the Riemann zeta function, Robustness of fictitious play in a resource allocation game, SAR imaging: An autofocusing method for improving image quality and MFS image classification technique.*

Among the contents of [63] and [64], mention the following specific issues: *Electric Machine Experimental Monitoring System Based on Labview Environment, A Fuzzy multicriteria methodology to assist interagency interaction in relief operations, Laser technology-General aspects – Suitable lasers for ballistic defense, Chaotic random bit generator realized with a microcontroller, Conical slot antenna for air and sea vehicle applications, Expertise, experience, real operational need: The key factors for max effectiveness and min cost in modern military operations, Reliability of information operations, On the continuous dependence of solutions of boundary value problems for delay differential equations, KERVEROS I: An unmanned ground vehicle for remote-controlled surveillance, Low observable principles, stealth aircraft and anti-stealth technologies, Consensus algorithms within the C4ISR architecture, Mathematical approach of electromagnetic interference analysis and safety radiation zones identification, The quantum theory in decision making, The RFID's Security with a critical view, Risk Assessment Techniques as Decision Support Tools for Military Operations, Modern radar techniques for air surveillance & defense, Dimensioning of electric propulsion motors for war-ships using finite element method, Current usage of Unmanned Aircraft Systems (UAS) and future challenges: A mission oriented simulator for UAS as a tool for design and performance evaluation, Operation level of war: a tool for planning and conducting wars or an illusion?, Army rapid fielding by optimizing order picking routes in warehouses with parallel aisles – implementation in a real case study,Developing an In-House IPMS for the Hellenic Navy Gunboats, Voice over Internet Protocol, Optimal operation of war-ship electric power system equipped with energy storage system, Load estimation for war ships based on pattern recognition methods, Challenges and objectives for the national cyber-security strategy beyond 2020.*

Quantum Computation / Cryptography

My work on Quantum Computation / Cryptography covers the research in the domains of *Topological Quantum Computations*, *Biholomorphic Codes and Cryptosystems* and the Edition of two (2) special issue in Cryptography

- **Topological quantum computations**

A conventional computer uses bits, which are classical two-state systems. On the contrary, a quantum computer uses quantum two-state systems (called qubits). A quantum computer could efficiently decrypt many of the cryptographic systems in use today, including the prime integer factorization problem or the related discrete logarithm problem[35] and represent data to perform operations with polynomial speedup (including quantum database search, finding collisions in two-to-one functions and evaluating NAND trees). Frequently, quantum computers offer a more than polynomial speedup over the best known classical algorithm have been found for several problems, including simulation of quantum physical processes from chemistry and solid state physics, the approximation of Jones polynomial, and solving Pell's equation. However, other existing cryptographic algorithms do not appear to be broken by these algorithms [36]. For instance, some public-key algorithms are based on problems other than the integer factorization and discrete logarithm problems, like the McEliece and Niederreiter

35 See P.W. Shor, Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer, SIAM Journal of Computing 26(1997), 1484-1509

36 See D.J. Bernstein, Introduction to Post-Quantum Cryptography. Introduction to Daniel J. Bernstein, Johannes Buchmann, Erik Dahmen (editors). Post-quantum cryptography. Springer, Berlin, 2009.ISBN 978-3-540-88701-0

cryptosystems based on a problem in coding theory [37]. Further, lattice-based cryptosystems are also not known to be broken by quantum computers, and finding a polynomial time algorithm for solving the dihedral hidden subgroup problem, which would break many lattice based cryptosystems, is a well-studied open problem [38]. The main problems in realizing a quantum computer are local errors, thermic noise and quantum decoherence.*Errors* are typically corrected in classical computers by keeping multiple copies of information and checking against these copies. With a quantum computer, however, the situation is more complex. If we measure a quantum state during an intermediate stage of a calculation to see if an error has occurred, we collapse the wave function and thus ruin the calculation. Remarkably, in spite of these difficulties, error correction is possible for quantumcomputers [39]. One can represent information redundantly so that errors can be identified without measuring information. However, the error correction process may itself be a little noisy. More errors could then occur during error correction, and the whole procedure will fail unless the basic error rate is very small[40]. Random errors are also caused by the interaction between the quantum computer and the environment. As a result of this interaction, the quantum computer, which is initially in a pure superposition state, becomes entangled with its environment. This can cause observable errors. Since we cannot measure the state of the environment accurately, information is lost. In other words, the environment has caused *decoherence*. It was universally assumed until the advent of quantum error correction that quantum computation is intrinsically impossible since decoherence-induced quantum errors simply cannot be corrected in any real physical system [41]. However, when error-correcting codes are used, the entanglement is transferred from the quantum computer to ancillary qubits so that the quantum information remains pure while the entropy is in the ancillary qubits. One of the greatest challenges is controlling or removing quantum decoherence. This usually means isolating the system from its environment as interactions with the external world causes the system to decohere. This effect is irreversible and is usually something that should be highly controlled, if not avoided. Decoherence times for candidate systems, in particular the transverse relaxation time, typically range between nanoseconds and seconds at low temperature [42]. These issues are more difficult for optical approaches as the timescales are orders of magnitude shorter and an often-cited approach to overcoming them is optical pulse shaping. Error rates are typically proportional to the ratio of operating time to decoherence time, hence any operation must be completed much more quickly than the decoherence time.If the error rate is small enough, it is thought to be possible to use quantum error correction, which corrects errors due to decoherence, thereby allowing the total calculation time to be longer than the decoherence time. However, the use of error correction brings with it the cost of a greatly increased number of required qubits. The number required to factor integers using Shor's algorithm is still polynomial, and thought to be between L and L^2, where L is the number of bits in the number to be factored; error correction algorithms would inflate this figure by an additional factor of L[43]. A very different approach to the stability-decoherence problem is to create a topological quantum computer with anyons, quasi-

37 See R.J. McEliece,A public-key cryptosystem based on algebraic coding theory, Jet Propulsion Laboratory DSN Progress Report 42–44, 114–116

38 See E. Knill, R. Laflamme and W.H. Zurek: Resilient quantum computation, Science279(5349)(1998), 342–345

39 See A,R. Calderbank and P.W. Shor, Good quantum error-correcting codes exist, Phys. Rev. A 54(1996), 1098–1105

40 See D. Aharonov and M. Ben-Or, Fault-Tolerant Quantum Computation With Constant Error, STOC (1997), 176-188

41 See P.W. Shor: Scheme for reducing decoherence in quantum computer memory, Phys. Rev. A52 (1995),2493-2496

42 D.P. DiVincenzo, Quantum Computation, Science270 (5234) (1995), 255–261. Available online atBibcode1995Sci...270..255D. doi:10.1126/science.270.5234.255

43 See M. I. Dyakonov, Is Fault-Tolerant Quantum Computation Really Possible?, in: Future Trends in Microelectronics. Up the Nano Creek, Université Montpellier (2006-10-14), Wiley, pp.4–18 (edited by S. Luryi, J. Xu, and A. Zaslavsky). Available online atarXiv:quant-ph/0610117

particles used as threads and relying on braid theory to form stable logic gates [44]. *A topological quantum computer is a theoretical quantum computer that employs two-dimensional quasiparticles called anyons, whose world lines cross over one another to formbraids in a three-dimensional spacetime (i.e., one temporal plus two spatial dimensions). These braids form the logic gates that make up the computer.The advantage of a quantum computer based on (quantum) braids over using trapped quantum particles is that the former is much more stable. When anyons are braided, the transformation of the quantum state of the system depends only on the topological class of the anyons' trajectories (which are classified according to the braid group): the smallest perturbations do not change the topological properties of the braids.* This is like the effort required to cut a string and reattach the ends to form a different braid, as opposed to a ball (representing an ordinary quantum particle in four-dimensional spacetime) simply bumping into a wall. *Thus, the quantum information which is stored in the state of the system is impervious to small errors in the trajectories*.

While the elements of a topological quantum computer originate in a purely mathematical realm, recent experiments indicate these elements can be created in the real world using semiconductors made of gallium arsenide near absolute zero and subjected to strong magnetic fields.

Anyons form from the excitations in an electron gas in a very strong magnetic field, and carry fractional units of magnetic flux in a particle-like manner. This phenomenon is called the fractional quantum Hall effect. The electron "gas" is sandwiched between two flat plates of gallium arsenide, which create the two-dimensional space required for anyons, and is cooled and subjected to intense transverse magnetic fields.

Topological quantum computers are equivalent in computational power to other standard models of quantum computation, in particular to the quantum circuit model and to the quantum Turing machine model. That is, any of these models can efficiently simulate any of the others. Nonetheless, certain algorithms may be a more natural fit to the topological quantum computer model. For example, algorithms for evaluating the Jones polynomial were first developed in the topological model, and only later converted and extended in the standard quantum circuit model.

In the paper *«Research directions and foundations in topological quantum computation methods»* ([30]), we give a short overview of the recent research perspectives and mathematical foundations in topological quantum computation theory. In particular, we are interested in braid representation theory, topological invariants of braids, approximation with braiding generators and quantum hashing with the icosahedral group.

More specifically, in [30], after a background presentation on quantum computation, including Dirac's bra-ket notation, quantum systems, state vectors (pure states, mixed states), quantum superposition, density matrices, measurements, observables, we give the definition of a quantum computer and investigate qubit systems, entangled quantum states and quantum gates (Hadamard gate, Pauli-X gate, Pauli-Y gate, Pauli-Z gate, swap gate, controlled gates, Toffoli-CCNOT gate, Fredkin-CSWAP gate) with special emphasis to the set of universal quantum gates. After doing this, we consider quantum computations (discrete Fourier transform (or DFT) formula, discrete-time Fourier transform (or DTFT) and recall basic concepts from elementary particles (anyons, i.e. *bosons and fermions*) of quantum mechanics by explaining topological interpretations of interchanging *several* identical particles in 2 and 3 spatial dimensions. To do so, we make use of the following assumption. Keeping in mind the above considerations, we are in position to describe how topological quantum computing works.

44 M. Freedman, A. Kitaev, M. Larsen and Z. Wang, Topological Quantum Computation, Bull. Amer. Math. Soc. (N.S.)40 (1) (2002). 31–38. Available online atdoi:10.1090/S0273-0979-02-00964-3

The output depends on the topology of the particular braiding produced by those manipulations. Small disturbances of the anyons do not change that topology, which makes the computation impervious to normal sources of errors.

Continuing in accordance with the foregoing, in [53], we give a quick introduction to the diagrammatic theory of braids and knots, using Reidemeister moves. (It is clear that *every braidcan be converted into a knot (or link) by forming the closure.*Alexander's theorem provides the converse statement: *Every knot or link can be represented as a closed braid.*) Successive application of Reidmeister moves gives *equivalent knotdiagrams*. A formal mathematical definition is that *two knots are equivalent if one can be transformed into the other via a type of deformation of*$\mathbb{R}^3$ *upon itself*, known as an *ambient isotopy*. Conversely, *two diagrams in the three dimensional space represent the same knot if and only if the diagrams are ambient isotopic through a sequence of Reidmeister moves.* A *knot invariant* with values in a set E is a function

$$f: \{knotsdiagrams\} \to E$$

such that if the knot diagrams D and D' are ambient isotopic through a sequence of Reidmeister moves then $f(D) = f(D')$. In other words, a knot invariant is a "quantity" that is the same for equivalent knots. In particular, a *knot polynomial* is a knot invariant that is a polynomial. A general pattern to produce knot invariants is to take any function $g: \{classesofequivalentknotsdiargams\} \to E$and, then, consider the functional composition$f = g \circ \rho$ where $\rho: \{knotdiagrams\} \to \{classesofequivalentknotsdiagrams\}$is the function associating to each knot diagram the equivalence class of this knot. Due to Reidmeister's theorem, any invariant can be obtained following this general pattern. Of course, the main problem for such a technical construction is to formulate (and understand!)ρ. To be more specific, let us choose $E = \{classesofequivalentknotsdiagrams\}$ and$f(D) = \rho(D)$. For this choice, the knowledge of ρ would render the corresponding knot invariant a very precious topological tool, since it would be equivalent to the knowledge of an efficient answer to the equivalent knot representation problem of two diagrams. Instead, we may simplify by taking relevant functions f that lose enough of information on knots represented by diagrams. In this direction, we give classic examples of knot invariants (*crossing number*, *gordian number* and *"three-color" invariant*).

Next, in [30], we turn to finitely generated braid groups, more specifically to the theoretical and intuitive (physical, topological and algebraic) presentation of Artin's braid group on n strands and we investigate *representations* of braid groups and invariant of knots (*Burau representation*, *representations from* $R-$*matrices*, *representations of the braid group and solution of the Yang-Baxter equation*, *relating Yang-Baxter equation with unitary R-matrices and universal gates*, H*ecke algebras representations of braid groups and polynomial invariants of knots*, *HOMFLY polynomial*, *Alexander polynomial*, *Jones polynomial*, *bracket polynomial*, *Lawrence-Krammer representation of braid groups* and *polynomial invariants of knots*).

The infinite braid group can have both one-dimensional and higher-dimensional representations. Abelian anyons correspond to the one dimensional case. Non-abelian anyons correspond to higher dimensional representations. The non-abelian anyons are characterized by D-dimensional Hilbert spaces, so that every set of N non-abelian anyons can be found in D^N orthogonal quantum states. It follows that every set of N non-abelian anyons can encode $Nlog_2D$ *qubits*$(D > 1)$. Therefore, the non-abelian anyons are of particular interest for our purposes.Of these, the Fibonacci anyons satisfy the simplest rule fusion, meaning that fusion of two Fibonacci anyons delivers a single Fibonacci anyon together after a quantum state that is trivial statistical. Recall that fusion rules are rules that determine the exact decomposition of the tensor product of two representations of a group into a direct sum of irreducible representations. Our next aim in [53] is to encode quantum information using only Fibonacci anyons (called Fibonacci because Nanyons span a Hilbert space of dimension D equal to the $N+1$ Fibonacci number).

To the purpose of universal quantum computation, in [53], we want now to approximate, at any given accuracy, any single-qubit gate using as generators N elementary braidings. By carrying out Bruce Force (BF) searches over braids with up to 46 elementary braid operations we typically find braids which approximate a desired target gate to a distance of 10^{-3}. Recall that a BF search allows to find the best weave (: braid in which only one quasiparticle moves) of a given length $L$to approximate every target gate. Any operation which can be carried out by a braid can also be carried out by a weave. The number of possible braids grows exponentially as $3^{L/2}$, while the time required is exponential in the length and calculations become cumbersome for $L > 40$. Although the accuracy grows exponentially as $3^{-L/6}$, the BF approach optimal solution is reached very slowly.

However, the brute force search is inefficient for long braids because it samples the whole $SU(2)$space with almost equal weight. To get a faster algorithm we must enhance the sampling near the target gate we want to approximate. In this way we can get a much faster algorithm that finds good approximations for arbitrary $SU(2)$gates (but in general not the optimal one). Thus, in [53], we explore topological quantum hashing with the finite icosahedral group and its algebra. The distribution of the distance between the identity and the so-obtained braids has an intriguing connection to the Gaussian unitary ensemble of random matrices, which helps us to understand how close we can approach the identity in this way, i.e. the efficiency of the hashing algorithm.

Biholomorphic codes and cryptosystems

In recent years there has been a great progress in the fields of Cryptology and Security. This progress is interdisciplinary and interactive with the frantic development of technology. Modern technology now provides very powerful tools to solve all quantitative measurable problems. And, conversely, modern science is directed in priority to exploring new methods which will exploit all the potential of modern technology. Particularly, in Cryptology and Security, the effectiveness of all the applications depends directly on to develop the technological infrastructure. For example, **the impending construction of powerful quantum computer is a big incentive to devise new methods of cryptanalysis, and it seems to be a matter of time the questioning or even the abandonment of many among the classic algorithms**.

In view of this suffocating dynamics and deadlock, it should be sought different approaches. These approaches should give new and utilitarian cryptology methods, which will fully exempt from intensive technological monitoring, at least in depth time. The first thought that comes to mind is to effect an overstepping or even a complete elimination of the discrete nature of classical cryptology structures. Indeed, a discrete structure accentuates all the technological possibilities of computational accuracy and of implementation of effective combinatorial schemes. Such distinct structures may be found in most cryptosystems where ordinary utilitarian cryptology structures consist of discrete sets. As examples, we can mention the sets of encoding rules, the rules for decoding the encrypted texts, the sets of possible decryption keys, the sets of rules encryption and the sets of decryption rules. All these sets are discrete.

Historically, the first overstepping of the discrete nature of classical cryptology structures was made by applying the methods of Chaos Theory in Cryptography.Already by the 1950s, Claude Shannon pointed out that the mechanisms of contraction and expansion of chaos could be exploited in Cryptology.After a thirty-year recession, during the 1990s, about 30 published papers gave various encryption algorithms focusing on analog circuits.After 2000, the

Chaos Theory became recognized in military applications and inaugurated the Crypstic from Lexicon Data Limited (http://eleceng.dit.ie/arg/downloads/Crypstic.zip).

Generalizing to this direction, it is reasonable to look for an efficient adaptation of the discrete structures of classical cryptology within a suitable constant environment, in such a way that the arranging to be computationally functional and free of any possibility of revocation due to the narrow technological tracking.The purpose of [48] is to present a study of physical adaptation for all the classical discrete structures of cryptology within the environment of complex variables.Thus, in the first section we define the biholomorphic rules of encoding and decoding into a simple connected domain of the complex plane, and we will give several relevant examples.According to these rules, the alphabet source is embedded into an initial simple connected domain of the complex plane $\mathbb{C}$, which is then transformed successively to other connected domains of $\mathbb{C}$. The choice of the successive simply connected domains, as well as of the number and type of the successive transformations are elements that characterize exclusively the respective coding. From these examples, it follows prominently that the method is computationally functional and free of any possibility of revocation due to the technological tracking. In the second section, we study the properties of biholomorphic rules for encoding and decoding. Further, in the third section we define the holomorphic cryptosystems, as well as the evolutionary holomorphic cryptosystems.In holomorphic cryptosystems, all plain texts are embedded into an initial domain of the complex space $\mathbb{C}^n$ which, by means of successive holomorphic mappings, is transformed respective times in other domains of $\mathbb{C}^n$, resulting in a final domain of $\mathbb{C}^n$.The initial and final domain of $\mathbb{C}^n$, as well as the number and type of the successive transformations are elements that characterize exclusively the respective encryption. Similarly, in the evolutionary holomorphic cryptosystems, all plain texts are embedded into an initial parameterized domain of the complex space $\mathbb{C}^n$which, by means of successive holomorphic mappings, is transformed respective times in other parameterized domains of $\mathbb{C}^n$, resulting in a final parameterized domain of $\mathbb{C}^n$. The initial and final parameterized domains of $\mathbb{C}^n$, as well as the number and type of the successive transformations are elements that characterize exclusively the respective encryption.The extra element of the parameterization gives the option of determining continuously variable cryptosystems, i.e. cryptosystems whose form depends on the time parameter (or any other real parameter).From the examples developed in the third section, it is clear that the definitions of the holomorphic cryptosystems and of the evolutionary holomorphic cryptosystems are computationally functional and free of any possibility of revocation due to the technological tracking.Finally, in the fourth section, we study basic properties of holomorphic cryptosystems and evolutionary holomorphic cryptosystems.

- **Edition of two (2) books and two (2) special issues in Cryptography**

Finally, it is commonly accepted that a broad spectrum of modern scientific research in Military Science is based on the application of *Cryptographic Methods*. In particular, mathematical thinking and methodology are widely applied, whereas the construction of relevant paradigms is at the heart of much scientific work.

In the framework of promoting Mathematics and its applications, the Hellenic Military Academy, the Department of Mathematics of the National and Kapodistrian University of Athens, and the Greek Mathematical Society are jointly organizing, under my presidency, the April 2012 conference on *Cryptography and its Applications in the Armed Forces*, held at the Hellenic Military Academy.

The issued under my supervision two special issues [60] and [61], as well as a book [65] contain selected contributions presented in this conference. Topics covered include *Cryptosystems*, *Cryptanalysis*, *Quantum Cryptography*, *Topological Quantum Computations,Hash Functions, Codes*, *Code Systems*, *Algorithmic Number Theory and Cryptography*, *the RSA Cryptosystem and Factorization of Integers*, *Public-Key Cryptosystems*, *Cryptosystems and Chaos*, *Automated Tools and Formal Methods*, *Transposition Systems*, *Cryptographs and Cipher Machines*, *Computer Security*, *Network Security and Information Security*. Among the contents of [60], [61] and [65], mention the following specific issues:*On the cryptographic long term security, matrix representation of cryptographic functions, encryption schemes based on Hadamard matrices with circulant cores, proof carrying code using algebraic specifications, algebraic attacks on stream ciphers: recent developments and new results, some aspects of group-based cryptography, Stealth aircraft tactical assessment using Stealth entropy and digital Steganography, image encryption scheme based on coupled chaotic systems, passive synchronization methodfor frequency hopping systems, efficient implementation of the bundle security protocol for DTN-enabled embedded devices, minimum key length for cryptographic security, research directions and foundations in topological quantum computation methods, hash addressing of the quasi-permanent key arrays in multilevel memory, a military real time locating system installation and utilization approach, numerical linear algebra methods in data encoding and decoding, accessible secure information society applications via the use of optimised cryptographic calculations, performance of Web services security mechanisms: analysis and evaluation, e-learning, fuzzy methods and sign language video to enhance teaching for hearing impaired, encryption and biometrics: context, methodologies and perspectives of biological data.*

Further, in a second Springer book ([69] we got together selected contributions including several applications in Cryptography of Modern Discrete Mathematics and Analysis, such as *Fixed Point Theorems in Generalized b-Metric Spaces, Orlicz Dual Brunn-Minkowski Theory: Addition, Dual Quermassintegrals, and Inequalities, Modeling Cyber-Security, Solutions of Hard Knapsack Problems Using Extreme Pruning, A Computational Intelligence System Identifying Cyber-Attacks on Smart Energy Grids, Recent Developments of Discrete Inequalities for Convex Functions Defined on Linear Spaces with Applications, Extrapolation Methods for Estimating the Trace of the Matrix Inverse, Moment Generating Functions and Moments of Linear Positive Operators, Approximation by Lupaș–Kantorovich Operators, Enumeration by e, Fixed Point and Nearly m-Dimensional Euler–Lagrange-Type Additive Mappings, Discrete Mathematics for Statistical and Probability Problems, On the Use of the Fractal Box-Counting Dimension in Urban Planning, Additive-Quadratic ρ-Functional Equations in Banach Spaces, De Bruijn Sequences and Suffix Arrays: Analysis and Constructions, Fuzzy Empiristic Implication, A New Approach, Adaptive Traffic Modelling for Network Anomaly Detection, Bounds Involving Operator s-Godunova-Levin-Dragomir Functions, Closed-Form Solutions for Some Classes of Loaded Difference Equations with Initial and Nonlocal Multipoint Conditions, Cauchy's Functional Equation, Schur's Lemma, One-Dimensional Special Relativity, and Möbius's Functional Equation, Plane-Geometric Investigation of a Proof of the Pohlke's Fundamental Theorem of Axonometry, Diagonal Fixed Points of Geometric Contractions, A More Accurate Hardy–Hilbert-Type Inequality with Internal Variables, An Optimized Unconditionally Stable Approach for the Solution of Discretized Maxwell's Equations, Author Correction to: Moment Generating Functions and Moments of Linear Positive Operators.*

Security

My work on Security is directed towards five (5) directions: *Mathematical description of cyber attacks, modeling cyber-security, security and formation of network-centric operations, information security, stochastic analysis of cyber attacks, epidemiological diffusion and discrete branching models for malware propagation in computer networks*

- **Mathematical Description of Cyber-attacks**

The purpose of [43] is to document a holistic modeling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. The starting point is to determine the concepts of valuations and vulnerabilities of parts of a node constituent. Based on these two concepts, one may be led to consider the fundamental concept of node supervision and subsequently to give the definition of cyber-effects and from this the definition of cyber-interaction. As we shall see a germ of cyber-attack can be viewed as a family of cyber-interactions with coherence properties and depending strongly on subjective purposes, information and/or estimates on the valuations and the vulnerabilities of parts of the involved nodes. In general the germs of cyber-attacks can be distinguished in three types: the germs of correlated cyber-attacks, the germs of absolute cyber-attacks and the germs of partial cyber-attacks. This approach provides immediate possibility of rigorous determination of the concepts of proactive cyber defense and proactive cyber protection.

- **Modeling Cyber-security**

The paper [45] documents a holistic modeling background and a corresponding mathematical theory to provide a rigorous description of cyber-attacks and cyber-security. After determining valuations and vulnerabilities of parts of a node constituent, we recall the definitions of cyber-effect and cyber-interaction. Based on these concepts, we give the mathematical definitions of cyber navigation and infected node and we explain what is meant by dangerous cyber navigation and protection of cyber nodes from unplanned attacks. Our discussion proceeds to a rigorous description of passive and active cyber attacks, as well as the relevant protections and concludes with a brief report on reasonable questions.

- **Security and Formation of Network -Centric Operations**

The paper [39] explores various concepts related to the Network Centric Warfare framework and investigates security and formation aspects of network centric operations. It is divided into 5 sections. The first section deals with definitions and background information of key terms such as Cyber Warfare, Information Warfare, C4ISR, and Network Centric. Special emphasis is given to Network Centric Operations (NCO) Conceptual Framework. The second section briefly reports and analyzes the three main thematic NCO-pillars: Net Centric Theoretical Foundations / Mathematical Modeling, Net Centric Technologies and Related Issues and Operational Experiences. Next, in the third section we apply graph theory concepts to NCO.

To do so, we consider Wong-Jiru's multi-layer graph model of NCO and we describe interlayer relationships. Our analysis proceeds with definitions and implications of several NCO-layered metrics (:out-degree, in-degree, density, reachability, point connectivity, distance, number of geodesics, maximum flow, network centrality, Freeman degree centrality, betweenness centrality, closeness centrality, edge betweenness, flow betweenness). The section ends with the mention of key advantages of the multi-layer NCO model. The fourth section investigates the security problem of network centric operations by applying methods of vertex pursuit games. Specifically, we suppose an intruder (or attacker) has invaded into the complex process of a Network Centric Operation with the intention to destroy or cause sabotage at the vertices of one or more of its five layers (:Processes, People, Applications, Systems, Physical

Network). The intruder could represent virus or hacker, or other malicious agents intent on avoiding capture. A set of searchers are attempting to capture the intruders. Although placing a searcher on each vertex of a layer guarantees the capture of the intruders, we discuss and investigate the more interesting (and more difficult) problem to find the minimum number of searchers required capturing the intruders. A motivation for minimizing the number of searchers comes from the fact that fewer searchers require fewer resources. Network Centric Operations that require a smaller number of searchers may be viewed as more secure than those where many searchers are needed. Finally, in section 5 we take up with the problem of network centric warfare strategic formation. After introducing distance-based operational utility functions, we keep to the study of two layer distance-based operational utilities and of best response NCO-graphs. Then, we consider pairwise operational stability in the network centric operations and we conclude with a study of the network centric operations formation with arbitrary operational utility functions.

- **Information security**

  In [33], afterstudying the stochastic process of cyber-attacks against a cyber system, we investigate the set of intermediate times between successive cyber-attacks. Then, we give basic definitions and properties on the expected number of cyber-attacks in a closed time interval and present renewal theorems, as well as a description of exact and asymptotic probability distributions for the main occurrence moments of cyber-attacks. Further, we outline stationary renewal processes of cyber-attacks. The paper concludes with asymptotic properties for the counting function of cyber-attacks.

- **Epidemiological diffusion and discrete branching models for malware propagation in computer networks**

  Today's enterprise systems and networks are frequent targets of malicious attacks that can disrupt, or even disable critical services. The spreading of malicious software (malware) has become one of the major issues in contemporary networking infrastructures, emerging at various levels and occasions. Significant work has been performed, however as the penetration of wireless ad hoc and sensor networks increases the interest of malware propagation in wireless networks increases as well.

  The paper [36] focuses on the study of epidemiological diffusion and discrete branching models for malware propagation in computer networks.

  As it is well known, the malware propagation in computer networksand communication presents many similarities with the spreading of diseases and biological viruses in living organisms. Towards this end, there have been serious efforts to model malware propagation using methods of epidemiology with appropriate modifications and the bringing together varying parameters respectively, to study the spread of malware on computer networks.

  In this direction, Section 1 contains a brief but comprehensive overview of well known epidemiological models for the diffusion of malware in computer networks. These models are continuous-time differential systems, Markovianprocesses and models based on the theory of closed queuing networks. In Section 2, we define infected nodes' trees and the associated Markovian discrete branching processes in discrete time, known as Bienaymé-Galton-Watson (BGW) malware propagation processes. Then, we give basic results, using generating functions and expectations of network nodes' infections, as well as of the recovery probability from infections. Further, we introduce BGW malware propagation processes with annexation and removal laws for network nodes and state three limit theorems. Next, in Section 3, we define the notion of quasi-stationarityof malware propagation for Markov chains and provide basic results in the case of a finite-state space. In the case of BGW malware propagation processes, we characterize Yaglom quasi-stationary limits of malware propagation (one-dimensional distribution conditional on non recovery from infection) and the Q-processes of malware propagation (process conditioned on non-recovery from infection in the distant future). In Section 4, we show how to code the genealogy of a BGW infected nodes' tree thanks to a killed random walk. The law of the total progeny of the infected nodes' tree is studied thanks to this correspondence. Alternative proofs are given via Dwass–Kemperman identity and the ballot theorem. Last, in Section 5, we introduce the coalescent point process of branching infected nodes' trees: on a representation of the quasi-stationary genealogy of an infinite set of infected network nodes, which is also doubly infinite in time; on splitting trees of malware propagation, which are those infected nodes' trees. Moreover, we study the malware propagation jumping chronological contour process of the splitting tree truncated up to a time t, which starts at the 'complete uselessness' time of the progenitor infected node, visits all existence times (smaller than t) of all infected nodes exactly once and terminates at 0. Finally, we investigate the coalescent point process of operational infected network nodes and we emphasize tothe rate at which the breadth process of operational infected nodesgrows exponentially on the event of non-recovery.

Mathematical Modeling

My work on Mathematical Modeling is directed towards seven(7) directions.

- **Mathematical Modelling of Cyberspace**

In [44], we give a mathematical definition of cyberspace including basic specifications of its different formalities. To this end, we set an appropriate framework for determining adequate theoretical background, allowing rigorous, supple, univalent and adaptive description of what exactly we mean by saying "cyberspace". At the basis of this framework is the concept of the $e-$category W_e. An $e-$categorycan be viewed as an infinite $e-graph(V,E)$ with vector weights, in such a way that the $e-$nodes in V are the $e-$objects, while the $e-$edges or $e-$arcs in E are the $e-$morphisms. Given this notion, we investigate the possibility of allocating vector weights to objects and morphisms of any $e-$category W_e. We also introduce a suitable metrizable topology on $e-$ graphs and $e-$ categories. Themost significantbenefits coming from the consideration of such a metric d_{W_e} in the set $ob(W_e)$ of objects of an $e-$categorycan be derived fromthe definitions of *cyber-evolution* and *cyber-domain*. Bearing all this in mind, we define the *local* $e-$*dynamics*, as a mapping $cy: [0,1] \to \left(|ob(W_e)|, d_{W_e}\right)$; its image is an$e-$*arrangement*. The points of an $e-$*arrangement* are the *instantaneous local* $e-$*node manifestations*. An $e-$arrangement together with all of its instant $e-$morphisms is an $e-$*regularization*. The elements of the completion $|ob(W_e)|$of the set $ob(W_e)$ of objects of an $e-$categoryare the *cyber-elements*, while the topological space $\left(|ob(W_e)|, d_{W_e}\right)$ is called a *cyber-domain*. A continuous local $e-$dynamics is said to be a *cyber-evolutionary path* or simply *cyber-evolution* of the cyber-domain. A cyber-arrangement together with its instantaneous homomorphisms is called a *cyberspace*. We investigate conditions under which an$e-$regularization may be susceptible of a projective $e-$limit. Subsequently, we define and discuss the concept of the *length* in a cyber-domain. The intrinsic cyber-metric is a metric possible to define on every cyber-domain. For this metric the distance between two cyber-elements is the lengthof the shortest cyber-track between these cyber-elements. We will conclude with a discussion about the *speed* of a cyber-evolution. Finally, we will give simple pointwise and uniform convergence of cyber-evolutions.

- **Mathematical models in Deterministic CombatTheory**

  Our second specialization of my work on Mathematical Modeling, is directed towards to the *deterministic combat models* ([82] and [83]).

  Many centuries passed until to form a proper scientific background and appropriate military knowledge to investigate the philosophical claim that *any kind of war is governed by principles and rules*. The true scientific background formed from systems of differential equations and data from many historical battles.

  The beginning was in 1902 by the American JV Chase, followed in 1914 by the Englishman F.W Lanchester and in 1915 the Russian M. Osipov. All these showed systems of differential equations describing relationships existing between loss and size of two opponent military forces. The various systems became known as "*Lanchester mathematical combat models*". After the second world war, began a intensive effort to verify and extend the original or adapted Lanchester mathematical combat models in as many historical battles.

  The effort began slowly to assist the great progress of computers. However, there were discouraging results and, for this reason, it was deemed appropriate to revise either by introducing probabilistic generalizations, or by enriching the models with extra relationships on qualitative or quantitative military variables (such as during a fight, surprise, ethnicity, ethics, leadership, level of military training, growth rate, overall promotion, human losses, losses in artillery, tanks and airplanes, etc.), or by a complete replacement of Lanchester mathematical combat models.

  In the two monographs [82] and [83] (:«Operations Research and its Military Applications. Introduction to the Stochastic and Renewal Theories of War. Volume 1. Issue 1», Hellenic Military Academy, Vari Attikis, Greece, 2007, pp.232+xiii and «Operations Research and its Military Applications. Introduction to the Stochastic and Renewal Theories of War. Volume 1. Issue 2», Hellenic Military Academy, Vari Attikis, Greece, 2007, pp.248+xiii), we arelaying all the fundamentalsof a deterministic combat models moderntheory.

- **Ellipsoid targeting with overlap**

  In [42], we investigated the possibility of destruction of a passive point target. Subsequently, we study the problem of determination of best targeting points in an area within which stationary or mobile targets are distributed uniformly or normally. Partial results are given in the case in which the number of targeting points is less than seven or four, respectively. Thereafter, we study the case where there is noinformation on the enemy distribution. Then, the targeting should be organized in such a way that the surface defined by the kill radii of the missiles fully covers each point within a desired region of space-time. The problem is equivalent to the problem of packing ellipsoids of different sizes and shapes into an ellipsoidal contained in R4 so as to minimize a measure of overlap between ellipsoids is considered.

- **Cyberspace Modelling and Proactive Security**

  In [43],we document a holistic modeling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. The starting point is to determine the concepts of valuations and vulnerabilities of parts of a node constituent. Based on these two concepts, one may be led to consider the fundamental concept of node supervision and subsequently to give the definition of cyber-effects and from this the definition of cyber-interaction. As we shall see a germ of cyber-attack can be viewed as a family of cyber-interactions with coherence properties and depending strongly on subjective purposes, information and/or estimates on the valuations and the vulnerabilities of parts of the involved nodes. In general the germs of cyber-attacks can be distinguished in three types: the germs of correlated cyber-attacks, the germs of absolute cyber-attacks and the germs of partial cyber-attacks. This approach provides immediate possibility of rigorous determination of the concepts of proactive cyber defense and proactive cyber protection.

- **Mathematical models in Portolio Analysis of Defense**

Allocation of large governmental budget proportions for national defense projects in several countries on a worldwide basis constitutes an apparent observation and strategic choice of diplomacy. Thus, serious enhancement of military decision making technologies is a matter of paramount importance.

Our purpose in [105] is to capture and classify all existing research activity concerning the application of portfolio analysis in military decision situations. We meticulously demonstrate actual and potential benefits of exploiting portfolio analysis in various types of military applications. The outmost aim is to document that the modeling framework of portfolio analysis offers the most solid methodological basis for resolving the inherently complex nature of most military decision problems.

- **Mathematical models in Economy**

In the paper *«Mathematics and Economy»* ([10]), we showed, in a few lines, that the qualitative approach through mathematical modeling, first, asks economic questions that are of interest from a mathematical point of view. Starting from a formal and general microeconomic model, we showed that the corresponding theoretical model reflects reality, and that the enhanced contribution of mathematical tools can provide answers that bear witness to the primary role of Mathematics in Economics.

- **Mathematical models in Fluid Mechanics**

Finally, in the monograph «Fonctions Holomorphes et Ecoulement Stationnaire Non Tourbillonaire d'un Fluid Incompressible Plan» ([97]), we examine emergence and role of holomorphic complex functions during the non-stationary planar outflow of an incompressible fluid. Inthe firstpart, after giving fundamental concepts of fluid mechanics (:stationary motion, incompressible fluid,the spinning motion), we consider the holomorphic conjugate velocity field of an incompressible fluid during its non-stationary spinning motion and treat the formed motion lines and the lines of equal potential. The second part investigates the uniform traffic flow around a disc into the planar fluid, while the third part studies the relationship between flow and conformal mapping. In this direction, the object of the fourth and last part is the description of stationary fluid dynamics via Joukowski's formula.

- **Edition of book in Mathematical Models of Military Logistics**

In the past few years, there has been an increased interest in the planning and execution of defence and combat logistics operations. Defence logistics is the basic supporter responsible for sourcing and providing nearly every consumable item used by military forces worldwide. The latter is also responsible for providing the Department of Defence and other governmental agencies with comprehensive solutions in procurement, demand forecasting, inventory control, warehousing, and transportation operations in the most effective and efficient manner possible.

To this end, in [67], we aimed to compile a reference book that highlights advanced Operations Research and intelligent methods, techniques, and state-of-the-art systems for tackling complex problems in military logistics. Topics covered include a wide spectrum of research issues ranging from *demand forecasting, location-allocation problems, and inventory control, to actual implementation, including resource allocation, replenishment tactics, and vehicle routing, all in the defence logistics area*. They also include issues related to *managing, sharing and processing information to enable an effective coordinated defence response*.

In addition, the edited volume includes *integrated case studies that describe the solution to actual military logistics problems of high complexity*. Among the contents of [67], mention the following specific issues: *World military expenditure, Defending and transporting military supplies in networks under partially strategic attacks, Reliability study and cost analysis of military operations: methods and applications, Intelligent methods for flocking routing of military formations, Transporting personnel in need of medical attention from the field to medical facilities, Multivariate statistical analysis and fuzzy analytical hierarchy process as strategic tools in the supplier selection procedure of military critical Items, Piracy phenomenon analysis using an agent based modeling approach, A combined inventory and lateral re-supply model for repairable items, The proposal of demand estimation of repairable items for the weapon systems during the initial support period: F-16 case study, A transportation planning tool for managing humanitarian & relief operations, A metaheuristic reconstruction algorithm for solving bilevel Vehicle routing problems with backhuals for army rapid fielding, Modelling and analysis of different localization of JSF squadron using causal maps, Using plane tessellation algorithms to optimize resource allocation, and UAV mission planning: from robust to agile.*

Big Data

My work on Big Data is directed towardsone(1) direction. Indeed, the aim of [46] is to document a quantitative systemic modeling for the processing of big data flow. Since, according to official calculations, the total global flow of data exceeds 150 million petabytes annual rate, or nearly 500 exabytes per day, it is very clear that the ever-increasing volume of data will soon cause great difficulty in the efficient processing of information and will make extremely difficult task of processing the data flow. In order to urgently overcome this obstacle, a good idea seems to be ***the appropriate choice of data amounts***.

To this direction, this paper studies a reasonable question which arises and may be constitute a central subject of discussion in subsequent additional scientific studies. The question relates to the *preference of choices and priorities in the processing of big data*. Equivalently, *if each one of a group of data processors prefers to be limited to different sets of data amounts from a collection of big data, then how much the different priorities of processing could lead to equilibrium situations or contrasts*? The motivation for answer this question is inspired by the economic theory of core and equilibriums, and, the relevant analysis has adapted on the framework for the mathematization of a large economy presented by W. Hildenbrand.

The paper is divided in two parts. The first part examines the case of a single data processor. Obviously, for each data entity in the domain of his competence, the processor can choose or use only an amount of data. Thus, in section 1.1, we describe how through its options, the data processor may prefer to focus only on some choices. A program of data selection for the processor specifies the data amount of each entity that the processor may take into account. Then, in Section 1.2, we study the selectivity display of a processor in order to actually exploit a certain amount of data from another. A *data selection preference* is the relation that determines such any selectivity. In order to establish a well such preference, in Section 1.3, we show how a processor should associate certain significance in each component of the system (vector) of times of data processing, while in section 1.4 we study the topology of the space of data selection preferences and we shall describe neighboring preferences of a given data selection preference. Having regard to all these, in the next section 1.5 we investigate the lower hemicontinuity of the relation defining the set of all *rational choices for the data amounts*, and in section 1.6 we deal with the concept of the *mean rational data amount choice for a set of data processors*. The second part of the paper is devoted to the case of several data processors. In this case, each of the processors has its own priorities and preferences, and, after a brief introduction, we see that there are cores and equilibriums of contrasts, the study of which may provide useful information (sections 2.3 and 2.4).

Data Management

My work on Data Management is directed towards one(1) direction. Indeed, the aim of the paper [55] is to document a quantitative systemic modeling of data management.

To this end, in [55] weinvestigate the reasonable question which arises and may be constitute a central subject of discussion in subsequent additional scientific studies. The question relates to the *subjectivity of choices and preferences in the management of big data*. Equivalently, if each one of a group of data managers prefers to be limited to different sets in a larger collection of big data, then when and how much the different preferences of the managers can lead to equilibrium situations or contrasts? Without loss of generality and in order to simplify the overall formulation of the model, we will assume continuously that *there is a complete objectivity in all data options and preferences*, in the sense that all data agents have agreed for the finalized selection of all weighted data. Moreover, for the same reasons, we will assume also regularly that *all data measurements were carried out with sufficient reliability* to such an extent as to preclude any discrepancy in the estimates of the predictions.

Prediction of Systemic Events

My work on Prediction of Systemic Events is directed towards two (2) directions. The investigation of a Deterministic Prediction Theory and, in particular, the mathematization of Systemic Geopolitics.

Indeed, in [49], we gave a general method for predicting spatio-temporal regions with "strange" systemic occurrences. To do so, we consider systemic indices and their measurements into the under consideration fixed spatio-temporal region. Given a suitably chosen family of preselected future points, the magnitude of the (Euclidean or not) distance between the surface of these systemic indices and a parametrized surface which interpolates or passes very close to the points of systemic measurements and given preselected vector values may be viewed as a measure for assessing the appearance of peculiar systemic incidents over the region under consideration, so, depending on these preselected points, we provide a general algorithmic framework for predicting spatio-temporal regions into which crucial systemic events are expected

Further, in [37] (and [38]) we discussed the mathematical foundations the geopolitical systemic modeling. Specifically, the aim of [37] is to document this holistic geopolitical systemic modeling. To this end, we will give two general mathematical models predicting geopolitical events in a geopolitical system. The starting point is to consider weighted geopolitical indices and their measurements. A weighted geopolitical index is a quantity which refers exclusively to a geopolitical entity at any point of the space-time, endowed with an associated threshold above and below which it is marked a geopolitical change in the conduct of the geopolitical system (e.g. distinct spatiotemporal historical phases). A geopolitical measurement gives the value of a geopolitical entity measured at some discrete time moments, in the same, homogeneous spatiotemporal historical phase, and some geographic location points. If we limit ourselves within a given region of space-time, then the corresponding set of weighted geopolitical indices over this region is said to be a universality of weighted geopolitical indicators. The magnitude of the (Euclidean or not) distance between such a universality of weighted geopolitical indices and a parameterized surface which interpolates the discrete points representing the values of a geopolitical measurement can be considered as a measure for assessing the occurrence or not of a geopolitical event. To this direction, we will demonstrate and give two general frameworks for determining the time moments and geographical location points at which is expected the appearance of peculiar geopolitical events. The corresponding algorithmic formulations show that the prediction problem is reduced to two respective classical nonlinear optimization problems.

Two basic and reasonable questions arise immediately and may be constitute the central subject of discussion in subsequent additional scientific studies. The first question relates to the *subjectivity of geopolitical choices and priorities*: given that it is very doubtful whether the considered set of weighted geopolitical indices could be considered as exhaustive, one wonders if the above prediction is ultimately reliable. Equivalently, if a geopolitical analyst considers a set of weighted geopolitical indices and if another geopolitical analyst considers a different set of weighted geopolitical indices, then how much the two predictions will differ or diverge? The second question concerns *the reliability of geopolitical measurements*: given that many geopolitical measurements are based on qualitative data, hierarchical structures and design practices, statistical methodologies, and information files, how much the reliability of the result of geopolitical measurements could affect the validity of a prediction? In other words, any divergences in the measurement values how much affect the accuracy of the time moments and of the locations where we expect appearance of a geopolitical event?

Without loss of generality and in order to simplify the overall formulation of the model, in what follows, we will assume continuously that *there is a complete objectivity in all geopolitical options and priorities*, in the sense that all geopolitical analysts have agreed for the finalized selection of all weighted geopolitical indices For the case in which it arises question concerning subjectivity of geopolitical analysts' preference priorities, the interested readers are referred to the [38]. Moreover, for the same reasons, we will assume also regularly that *all geopolitical measurements were carried out with sufficient reliability* to such an extent as to preclude any discrepancy in the estimates of the predictions.

In the first two sections, we will provide basic definitions of the geopolitical concepts which will be used subsequently, such as the weighted index geopolitical and the geopolitical measurement. Moreover, the development we adopt in the second section for the concept of geopolitics measurement is fully aligned to a summarized description of the classic analysis of measurement systems.

In the third section, we will give algebraic formulations that refer to the geopolitical space of weighted geopolitical indices over a given system. The algebraic approach will give the possibility of introducing new concepts, such as the concept of the geopolitical fiber at a time and in one geographical location and the concept of the geopolitical affinity between two geopolitical systems. This approach within the third section will come with the consideration of a fiber product between two spaces consisting of weighted geopolitical indices.

In the fourth section we will introduce and examine the structure of the, so-called, universalities of weighted geopolitical indices. We will distinguish between two cases: the case in which such a given universality forms a parameterized surface into the geopolitical space of weighted geopolitical indices over a given system and the case where the universality exhibits discontinuities. In the same section, we will describe the aspect of a geopolitical measurement at some discrete time moments and some geographic location points, and then we will discuss the concept of deviation which can have such a geopolitical measurement from a given universality of weighted geopolitical indices.

Based on this background, in the fifth, and final, section we will move on to considering a distance between a given universality of weighted geopolitical indices and the parameterized surface which interpolates the points that represent the values of the corresponding geopolitical measurement. Notice that *the choice of an appropriate distance is non-unique, and may be determined according to the formulation of each problem*. This approach allows predicting of time moments and of geographical location points at which is expected to happen a geopolitical event. Indeed, *if at some point of space-time, the distance between the two surfaces exceeds a given critical value, then it means that at the prescribed time moment is expected a geopolitical event in this location point*.

This prediction will be described in two cases. Firstly, in the case where the measurements are conducted at discrete time moments and the geographical location remains constant. In such a case, the parameterized surface which interpolates the points of the measurement is defined by means of the Lagrange unique polynomial. Secondly, in the case where the measurements are conducted at discrete time moments and (generally) over different location points. In such a case, the parameterized surface which interpolates the points of the geopolitical measurement is defined by means of the Kergin unique polynomial. In both cases, the methods, which lead to the prediction of a geopolitical event, are given by two corresponding algorithms, resulting in two constrained nonlinear optimization problems which can be solved by one of the beautiful methods of the relevant literature. Notice that the choice of the interpolation method giving the parameterized surface is not binding. For example, one can use spline functions, instead of interpolation polynomials, but the central idea of the method remains unchanged.

Diophantine Equations

My work on Diophantine Equations is directed towards two (2) directions. The proof of Beal's Conjecture and the proof of non-existence of non trivial solutions to the Diophantine equation $Ax^p \pm By^q = Cz^r$.

In the early twentieth century, David Hilbert presented twenty-three great mathematical problems which featured main directions of scientific research throughout the period that followed [45] [46]. By analogy, in 2016, John F. Nash and Michael Th. Rassias gave a list with seventeen, currently unsolved problems in modern mathematics, in the belief that these problems are expected to determine several of the main research directions at least during the beginning of the XXI century [47]. The third problem in the series of this list refers to the exploration of the possibility of extending famous Fermat's Last Theorem.

Even before achieving a proof of this Theorem[48] , various generalizations had already been considered, to equations of the shape $Ax^p + By^q = Cz^r$, for fixed integers A, B and C.In this direction, the Theorem of Henri Darmon and Andrew Granville states that *if A, B, C, p, q and r are fixed positive integers with $p^{-1} + q^{-1} + r^{-1} < 1$, the equation $Ax^p + By^q = Cz^r$* has at most finitely many solutions in coprime non-zero integers x,y and z [49]. So far, apart this Theorem, only a few results have been obtained in this general case.Nevertheless, in the special case $A =$

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$B = C = 1$, has been made more progress, and more specifically,as made clear through those mentioned by Michael A. Bennett, Imin Chen, Sander R. Dahmen and SorooshYazdani [50] [51], except the solutions identified by PredaMihăilescu in the Catalan equation [52] and the solutions derived from some elementary numerical identities where at least one among the exponents p, q and r equals 2, in all other known cases, there is no non-trivial solution of this equation, once it is assumed that $A = B = C = 1$ and p, q, r ≥ 3 [53] [54] [55] [56] [57] [58] [59] [60] In this connection, Beal's Conjecture argues that if$\min\{p, q, r\} \geq 3$, there areno non-trivial co-prime integral solutions to the generalized Fermat equation$x^p + y^q = z^r$ [61] [62].So far, many computational attempts have produced strong indications that this conjecture may be correct [63]. The aim of [47] is to give a constructive proof of this Conjecture. The main purpose of the [51] and [52] is to extend the proof adopted in [47] to the most general case of the Diophantine equation $Ax^p \pm By^q = Cz^r$, with $gcd(Ax, By, Cz) = 1$, by showing that *this equation has only trivial integer solutions whenever his exponents* p*,* q*and* r *are all greater than two*. In particular, we reaffirm Bell's Conjecture and give a brief and new proof of Fermat's Last Theorem.

Addtive Number Theory

In [53], we study the principle of symmetric positioning of primes around the natural numbers and relying on this we turn to a proof of the Binary (or Even or Strong) Goldbach's Conjecture and, subsequently, of Ternary (or Odd or Weak) Goldbach's Conjecture.

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SUMMARY TABLES SHOWN SPECIFIC ISSUES AREAS AND RELATIONSHIPS OF MY WORK

Numerical Analysis	Rational approximation	[1],[2], [6], [8], [9], [12], [13], [15], [16], [17], [18], [19], [24], [28], [35] and [76]
	Rational interpolation	[9]
	Best rational approximation and interpolation theory	[18]
	Rational approximation to vectors of mutually irrational numbers	[41]
	Orthogonal polynomials and numerical reconstruction of signals	[24], [28]
	Markov-type inequalities in multivariate complex approximation	[16], [19], [35]
	Acceleration of computational schemes	[9], [76]
	Interpolation methods for the numerical evaluation of finite Baire measures	[13]
	Multidimensional logarithmic residue formulasfor solving systems of non linear equations	[56], [57]
	Complex extrapolated successive overrelaxation	[11]
	Numerical solving of differential equations and integral equations	[22], [13]
	Numerical evaluation of integrals and derivatives	[9], [76]
	Markov-Type Inequalities with Applications in Multivariate Approximation Theory	[35]
	Writings (: Elementsof NumericalAnalysis, MathematicalProgramming, Numerical Optimization, Quantitative Complex Approximation)	[86], [87], [88], [77], [78], [79], [98]

Table 1

Complex Analysis	Universal power series in $\mathbb{C}$ and $\mathbb{C}^n$	(25], [29], [40], [50].
	Summability transforms and analytic continuationin $\mathbb{C}^n$	[1], [2], [4],
	Biholomorphic maps in $\mathbb{C}^n$	[14],[17]
	Integral representations in $\mathbb{C}$ and $\mathbb{C}^n$	[17], [29]
	Writings (:Complex Analysis and Geometric Analysis, Contact Geometry, Projective and Descriptive Geometry Topology and Differential Calculus)	[95],[96], [99], [100], [101], [102]

Table 2

History of Mathematics		
	History of continued fractions. Connection to rational approximation theory	[9]
	History of deterministic mathematical combat theories	[23]

Table 3

Stochastic Modeling and Numerical Simulation in Operations Research		
	Reliability of military operations	[107]
	Stochastic and renewal combat models	[20], [21], [82], [83], [84], [85]
	Missile allocation strategies	[72], [73], [92]
	Target coverage	[72], [91]
	Optimal placement of defense forces	[32], [105]
	Strategic defense	[84], [85]
	Numerical multi-agent simulation	[73]
	Writings (:Probability in military operations research)	[90]
	Edition of books and special issues	[58], [59], [63],[64]

Table 4

Quantum Computation / Cryptography		
	Topological quantum computations	[30]
	Biholomorphic codes and cryptosystems	[48]
	Edition of two (2) books and two (2) special issues in Cryptography	[60], [61], [65],[69]

Table 5

Security		
	Mathematical description of cyber attacks	[70]
	Modeling cyber-security	[76]
	Security and formation of Network-Centric Operations	[85]
	Information security	[86]
	Epidemiological Diffusion and Discrete Branching Models for Malware Propagation in Computer Networks	[88]

Table 6

Mathematical Modelling		
	Mathematical modelling of Cyberspace	[44]
	Mathematical models in deterministic combat theory	[82],[83]
	Ellipsoid targeting with overlap	[42]
	Mathematical models in portfolio analysis of Defense	[105]
	Mathematical models in economy	[10]
	Mathematical models in fluid mechanics	[97]
	Cyberspace Modelling and Proactive Security	[43]
	Edition of book in Mathematical Models in Military Logistics	[67]

Table 7

Big Data		
	Selective properties of big data	[46]

Table 8

Data Management		
	Selective properties of big data	[55]

Table 9

Prediction of Systemic Events	Deterministic Prediction Theory	[49]
	Prediction of geopolitical events	[37], [38]

Table 10

Diophantine Equations	Proof of Beal' s Conjecture	[47]
	Unsolvability of the Diophantine Equation $\boldsymbol{Ax^p} \pm \boldsymbol{By^q} = \boldsymbol{Cz^r}$ / Non-Existence of Non-Trivial Solutions to the Diophantine Equation $\boldsymbol{Ax^p} \pm \boldsymbol{By^q} = \boldsymbol{Cz^r}$	[51, 52]

Table 11

Additive Number Theory	Prime Numbers' Symmetric Distribution / Reflective Arrangement of Prime Numbers and Proof of Goldbach's Conjecture	[53, 54]

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http://books.google.gr/books?id=aYaAinJn2B4C&pg=PR5&lpg=PR5&dq=Applications+of+Mathematics+and+Informatics+in+Military+Science&source=bl&ots=fEBI0IwoEZ&sig=uC6XPk8KUlkS2EzAlRCht6Tp9v0&hl=el&sa=X&ei=A_07UrbAK4z24QT0rYH4AQ&ved=0CE8Q6AEwBQ#v=onepage&q=Applications%20of%20Mathematics%20and%20Informatics%20in%20Military%20Science&f=false)

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Theaching Statement

AnalysisofNicholasJ. Daras' Teaching

Teaching Philosophy

The most important thing that we can teach our students is that Mathematics is interesting, relevant, and fun. A student who is curious and interested in the subject is easy to teach, but unfortunately not all students arrive in the classroom in this state of mind. It is the responsibility of the teacher to present the subject in an interesting and engaging manner that shows the elegance and beauty of Mathematics as well as its applicability to solving concrete and real-world problems and to nurture each student's latent desire to learn.

It is also important to teach students how to approach the subject. This is especially true for introductorylevel courses. Introductory Mathematics is somewhat unusual in that we expect the students to learn general concepts such as abstraction, functional decomposition, and object-oriented design, while at the same time requiring that they internalize the syntax and often archaic details of their first programming language.

It is the responsibility of the teacher to balance and differentiate these two tasks, so that the students always understand that the programming language they are learning is merely one of many ways to express the higher-level concepts. New language constructs or programming techniques should be introduced to illustrate and implement higher-level concepts, and never vice versa.

In higher-level courses, I believe that the emphasis should be on collaboration, planning and design, and critical review. Complex scientific and engineering projects are rarely the work of an individual; students must learn to organize and work as teams as early as possible. Similarly, planning and design are essential elements of any large project, but are difficult skills to master (but are often given little weight in the curriculum). Finally, the ability to intelligently and objectively critique the work of others is a necessary precursor to being able to accurately evaluate one's own work.

Teaching Style

My personal style of teaching is based on the following principles:

Engage the students. Students must be active participants in the learning process, rather than passive observers. This is particularly important for lecture courses.

Establish fair and clear grading policies. Despite our best efforts to inspire students to learn simply for the joy of learning, there will always be many students who focus primarily on whatever aspects of the material they believe will result in their receiving a good grade. However, this is not always a bad thing . the proper grading and assessment policies can guide these students to focus their attention on the essential points.

It is also important that grading policies be fair and relevant to the objectives of the course; few things are more discouraging to students than receiving a low grade for work that they believe is good. Grading standards must also be flexible so that unconventional or original solutions are not penalized simply because they do not match the anticipated solution.

Set clear and realistic goals. Students respond best to goals that are both challenging and achievable. Forexample, extremely easy assignments are boring, allow students to become careless, and do not give the students any sense of accomplishment. In contrast, excessively difficult assignments are frustrating and intimidating. Unclear or ambiguous assignments are even worse because the students are apt to waste their time solving the wrong problem (and justi_ably resent the poor marks they receive when they fail to correctly guess what problem they were supposed to solve).

Identify and fix misconceptions early. Once a misconception takes root, it is dif_cult to remove. Waiting until the next assignment or test has been graded to discover that students are confused is a grave mistake.

Let the students make mistakes. Learning what doesn't work is just as important as learning what does. Students learn more from understanding *why* an incorrect answer is wrong than from simply memorizing the correct answer. Experimentation is essential to education; students must be encouraged to learn from their mistakes.

In fact, I even encourage my students to make mistakes. For example, I show them that adding intentional syntax errors to a working program is a good way to learn what error messages the compiler will generate in response so they can recognize those messages when they see them in the future.

Always respect the students. A teacher must respect the goals, needs, and individuality of each student and help each student do his or her best to achieve these goals. Not all students respond to the same methods, come from the same background, or have the same level of preparation. For example, one mistake that I made in my first year as a teaching assistant was to use metaphors and idioms that were meaningless to many of my international students. This is a very easy mistake to make and quite awkward to repair.

Teachers must also respect that students have other interests and engage in time-consuming activities outside of the classroom; there are limits to how much time students can reasonably be expected to spend on one course.

I have found that a good way to engage students and identify misconceptions quickly is to give very short quizzes during class following the discussion of each major idea. These quizzes are ungraded because they primarily test how well I am explaining the material and holding the attention of the students, and it is inappropriate for the students to receive poor marks if my exposition is ineffective or boring. Even though they are ungraded, these quizzes do motivate and focus the students on the key points of each class because they know that I expect them to be able to answer the questions correctly. If they cannot, it is a warning sign that they have not mastered the material and will have dif_cutly with their tests and assignments.

One of the crucial elements of engagement is that students must have the freedom and means to tell the teacher when they are lost or confused. In addition to quizzes, I use short questionnaires to get immediate feedback about the reading and my lectures. The key questions I ask are .Did I explain anything in a way that you found unclear or confusing?. and .Are there any topics you would like me to revisit during the next class?. Unlike my quizzes, which I try to incorporate into every class meeting, I use these questionnaires primarily when I am trying a new set of examples or teaching a subject for the first time.

Course Development

I consider creating new assignments to be the most interesting and rewarding part of course development, as well as the most challenging and important. Good assignments must be interesting and relevant in order to engage the students, and they must match the ability and background of the students. Assignments must also be written in a manner that explains clearly and unambiguously what the students are expected to do and how their answers will be evaluated. In most cases, assignments must also provide some amount of guidance about how the concepts the students have been learning can help them to do the assignment.

Similarly, I have found that the best way to organize and structure my lectures and writing is to begin by asking myself what questions I want to enable the students to answer. I usually begin with a very specific question (*e.g.* .How can I prove that the complexity of mergesort is $O(nlogn)$?.), and then generalize (*e.g.*.What general methods are there for solving the recurrence relations that describe the complexity of divideand-conquer algorithms?). I then attempt to answer the question, using only knowledge and intuition that I expect the students to have. Each false start, stumbling block or impasse is a topic I must address in lecture or the reading. At the same time, however, I am always careful to remember the higher goal of teaching, which is not to prepare the students to answer the questions we pose to them today, but to answer questions we have not yet imagined.

Teaching Experience

UNIVERSITY OF AEGEAN (1990-1992)

DURATION	COURSE	COURSE DESCRIPTION
1990-1991 (*Assistant Professor, Department of Mathematics*)	Multivariate Complex Analysis	*Subharmonicity and its applications, convexity, domains of holomorphy, pseudoconvexity, Hörmander's solution of the d-bar equation, Cousin problems, cohomology, and sheaves, the zero set of a holomorphic function, constructive methods, integral formulas for solutions to the d-bar problem and normestimates, holomorphic mappings and invariant metrics, manifolds, area measures, exterior algebra, vectors, covectors, and differential forms analytic varieties, CR manifolds*
1991-1992 (*Assistant Professor, Department of Mathematics*)	Complex Analysis	*Algebra of complex numbers – geometric representation of complex numbers –stereographic projection – various types of differentiation in the complex field, Cauchy Riemann equation, one point compactification and the Riemann sphere, analytic(holomorphic) functions – power series – linear fractional transformation – exponential – logarithmic and trigonometric functions, conformal mapping, definition and properties, elementary conformal mappings, complex integration – Cauchy theorem – general form of Cauchy theorem – Cauchy integral formula, Morera's theorem, Liouville theorem – fundamental theorem of algebra – Taylor's theorem – open mapping theorem – maximum modulus theorem – Schwartz lemma, singularities – Taylor and Laurent series expansion – Weierstrass theorem – Residue theorem – argument principle – Rouche's theorem, evaluation of standard type of integrals using residues.*
1991-1992 (*Assistant Professor,*	Numerical Analysis	*Solution of algebraic and transcendental equations,*

Department of Mathematics)		*bisection method, iteration method, method of false position, Newton Raphson method, solution of linear system of equations, matrix inversion method, gauss Jordan elimination method, Gauss Seidel iteration method, Cholesky LU decomposition method, power method for eigen values. Numerical interpolation, Newton forward and backward formula, Lagrange interpolation formula, Hermite interpolation formula, Newton divided difference formula, central difference formula, numerical differentiation and numerical integration, traphezoid rule, Simpson rule, double integration, numerical solution of differential equations, Taylor series method, Picard method, Euler method, Runge-Kutta method, predictor corrector method, Adam and Milne method.*

HELLENIC AIR FORCE ACADEMY (1994-2011)

DURATION	COURSE	COURSE DESCRIPTION
1994-2000 and 2001-2012 (*Professor ofMathematics withprivate law contracts)*	Mathematics *(annual course)*	*Equations of lines,inequalities, signs of factored expressions, functions including the definitions and properties of absolute value, power, polynomial, rational, trigonometric, exponential, and logarithmic functions, composition of functions,definitions and calculational methods for limits, horizontal and vertical asymptotes, continuity, intermediate value theorem, derivative, definition and geometrical interpretation, derivative as rate of change; velocity and acceleration, rules of differentiation, differentiation formulas for power, trigonometric, exponential and logarithmic functions, chain rule, Implicit differentiation, linear approximation to a differentiable function, maxima and minima; extreme value theorem; mean value theorem, increasing and decreasing functions, concavity, first derivative test; second derivative test, curve sketching, applied maximum - minimum problems, antiderivatives; integration formulas, area, definite integral, fundamental theorem of calculus, integration by substitution, inverse functions, inverse trigonometric functions, techniques of integration, numerical integration, improper integrals, applications of integrals (area, volumes), Taylor polynomials, differential equations: separable, linear first and second order, constant coefficients, undetermined coefficients, variation of parameters, sequences, infinite series, power series, Taylor series. Review of: vectors in* R^2 *and* R^3*, lines, planes, cross and dot products, and matrices, curves in* R^2 *and* R^3*, polar coordinates, parametrization, arclength, functions of several variables, limits and continuity, partial derivatives, chain rule, directional derivative, implicit functions.Vectors and Euclidean space. Functions of several variables: level curves and surfaces, limit and continuity, differentiation:differentiability, partial derivatives and the chain rule, directional derivatives, higher derivatives, applications: tangent planes, extrema, Lagrange multipliers, inverse function theorem and implicit function theorem,*

		differentiation, implicit differentiation, double and triple integrals, iterated integrals, double integrals in polar coordinates, triple integrals in cylindrical and spherical coordinates, change of variables, Jacobians, vector fields, line integrals, independence of path, Green's theorem, surface integrals, curl and divergence, divergence theorem, Stokes' theorem.Sequences and series of functions; pointwise and uniform convergence, Weierstrass M-test, differentiation and integration of series, power series, step functions and their integrals; integration of limits of increasing sequences of step functions, the Lebesgue integral and its basic properties, sets of measure zero, the monotone and dominated convergence theorems; Fatou's lemma, functions defined by integrals and differentiation under the integral sign, Fubini's theorem, square-integrable functions; completeness of L^2 *; Hilbert space axioms, the Hilbert space* l^2 *; Fourier series as an isometry of* L^2 *with* l^2*, self-duality of Hilbert spaces, the Fourier series of a function, Parseval's formula; the Riesz-Fischer theorem, the* L^2 *-density of trigonometric polynomials, Riemann-Lebesgue lemma, pointwise convergence of Fourier series, the Fourier transform and its properties; the Fourier integral theorem, convolution and the Fourier transform, the Laplace transform, applications to differential equations, further topics, e.g., the Dirac delta function and its Fourier transform (time permitting).*
1994-2000 and 2001-2011 (*Professor ofMathematics withprivate law contracts*)	Descriptive Geometry	*Orthographic projections, primary auxiliary views, lines, planes, successive auxiliary views, piercing points, intersection of lines, angle between planes, parallelism, perpendicularity, angle between line and oblique plane, mining and civil engineering, revolution, concurrent vectors, plane tangencies, intersection of planes and solids, developments, intersections of surfaces, shades and shadows*
1998-2000 (*Professor ofMathematics withprivate law contracts*)	Applied Mathematics (Numerical Analysis)	*Numerical solution of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; interpolation: error of polynomial interpolation, Lagrange, Newton interpolations, numerical differentiation, numerical integration: trapezoidal and Simpson rules, Gauss Legendrequadrature, method of undetermined parameters, least square polynomial approximation; numerical solution of systems of linear equations: direct methods (Gauss elimination, LU decomposition), iterative methods (Jacobi and Gauss-Seidel); matrix eigenvalue problems: power method, numerical solution of ordinary differential equations: initial value problems: Taylor series methods, Euler's method, Runge-Kutta methods.*

1998-1999 (*Professor ofMathematics withprivate law contracts*)	Operations Research	*Introduction to Operations Research (OR), introduction to foundation mathematics and statistics, Linear Programming (LP), LP and allocation of resources, LP definition, linearity requirement, maximization then minimization problems.graphical LP minimization solution, introduction,*

		simplex method definition, formulating the simplex model. Linear Programming – Simplex Method for maximizing. Simplex maximizing example for similar limitations, mixed limitations. Example containing mixed constraints, minimization example for similar limitations. Sensitivity analysis: changes in objective function, changes in RHS, the Transportation Model. Basic assumptions. Solution methods (feasible solution: the Northwest method, the lowest cost method. Optimal solution: the stepping stone method, modified, distribution (MODI) method). The assignment model:- basic assumptions. Solution methods:different combinations method, short-cut method (Hungarian method). MSPT: the Dijkestra algorithm, and Floyd's algorithm (shortest route algorithm).

NATIONAL AND KAPODISTRIAN UNIVERSITY OF ATHENS (2001-2011)

DURATION	COURSE	COURSE DESCRIPTION
2001-2011 (*Unsalaried Professor*)	Numerical Optimization *(graduate course)*	*Introduction - general concepts, simple examples, global vs local, gradient vs nongradient, black box vs intrusive methods. Unconstrained optimization - overview of algorithms, optimality conditions, solution techniques. Globalization - line search methods, trust region methods. Conjugate Gradient - linear, nonlinear, basic properties. Newton methods - Hessian calculation, line search, trust region, convergence issues, quasi-newton, BFGS, SR1, Broyden, limited memory BFGS, Gauss Newton methods, nonlinear issues. Constrained optimization - optimality conditions, nonlinear cases. Primal-dualinteriorpointmethods, main types ofprimal-dual algorithms,methods followingthe path (algorithm SPF,algorithmPC,algorithmLPF),potentialreductionmethods(algorithmPR), notfeasibleinteriorpoint methods (algorithmIPF,implantationproblem,homogeneousself-dual formulation) algorithmMPC. Quadratic programming - range and null space methods, active set methods for inequalities, penalty methods, sequential quadratic programming. Advanced topics - implementation, parallelization, interfaces, applications.*

2001-2002 (*Unsalaried Professor*)	Numerical Analysis	*Iterative solution of non-linear equation in one variable, Solution of linear systems: direct elimination, matrix factorization, iterative methods, matrix norms, convergence, iterative solution of non-linear systems by fixed point iteration and Newton's method, the symmetric eigenvalue problem, Gershgorin's theorem, the power method. Systems of linear equations, Gauss-Jordan elimination, homogeneous systems, rank, Vectors in* R^2 *and* R^3*, dot and cross products, projections, lines, planes, area, volumes, matrix transformations in* R^2*, linear transformations, matrix algebra, transpose, inverses, applications to systems of equations, determinants by row reduction and their properties, application to inversion,*

		area, eigenvalues, eigenvectors, diagonalization, polar coordinates, complex numbers, selected applications (Markov chains, economic models, least squares approximation, linear programming). Vector spaces, subspaces, independence, basis and dimension, row and column space of a matrix, rank, applications, linear transformations, kernel and image, composition, linear functionals, the double dual, transpose of a linear transformation. Orthogonality, Gram-Schmidt process, orthogonal diagonalization and least squares approximation, quadratic forms, SVD. Change of basis.

TECHNOLOGICAL EDUCATIONAL INSTITUTE OF HALKIDA (2002-2004)

DURATION	COURSE	COURSE DESCRIPTION
2001-2004 (*Professor on contract*)	Mathematics I	*Limits and continuity, derivatives and their basic properties, rules of differentiation, the mean value, theorem, L'Hospital's rule, application of derivatives: related rates, graphing, maximization, linearization, Newton's method, antiderivatives, the definite integral, the fundamental theorem, substitutions, exponentials and logarithms, inverse functions, growth and decay, inverse trigonometric functions, hyperbolic functions, applications of integration: areas, volumes of revolution, volumes by slicing, arc length, surfaces of revolution.*
2002-2003 (*Professor on contract*)	Numerical Analysis	*Iterative solution of non-linear equation in one variable, solution of linear systems: direct elimination, matrix factorization, iterative methods, matrix norms, convergence, iterative solution of non-linear systems by fixed point iteration and Newton's method, the symmetric eigenvalue problem, Gershgorin's theorem, the power method. Systems of linear equations, Gauss-Jordan elimination, homogeneous systems, rank, vectors in R^2 and R^3, dot and cross products, projections, lines, planes, area, volumes, matrix transformations in R^2, linear transformations, matrix algebra, transpose, inverses, applications to systems of equations, determinants by row reduction and their properties, application to inversion, area, eigenvalues, eigenvectors, diagonalization, polar coordinates, complex numbers, selected applications (Markov chains, economic models, least squares approximation, linear programming). Vector spaces, subspaces, independence, basis and dimension, row and column space of a matrix, rank, applications, linear transformations, kernel and image, composition, linear functionals, the double dual, transpose of a linear transformation. Orthogonality, Gram-Schmidt process, orthogonal diagonalization and least squares approximation, quadratic forms, SVD. Change of basis.*
2002-2003 (*Professor on	Applied Mathematics	*Inverse functions, inverse trigonometric functions, techniques of integration, numerical integration, improper*

contract)		*integrals, applications of integrals (area, volumes), Taylor polynomials, differential equations: separable, linear first and second order, constant coefficients, undetermined coefficients, variation of parameters. Sequences, infinite series, power series, Taylor series, review of: Vectors in* R^2 *and* R^3*, lines, planes, cross and dot products, and matrices, curves in* R^2 *and* R^3*, polar coordinates, parametrization, arclength, functions of several variables, limits and continuity, partial derivatives, chain rule, directional derivative, implicit functions.*
2003-2004 (*Professor on contract*)	Differential and Integral Calculus	*Vectors and Euclidean space, functions of several variables: level curves and surfaces, limit and continuity, differentiation: differentiability, partial derivatives and the chain rule, directional derivatives, higher derivatives, applications: tangent planes, extrema, Lagrange multipliers, inverse function theorem and implicit function theorem, differentiation, implicit differentiation, double and triple integrals, iterated integrals, double integrals in polar coordinates, triple integrals in cylindrical and spherical coordinates, change of variables, Jacobians, vector fields, line integrals, independence of path, Green's theorem, surface integrals, curl and divergence, divergence theorem, Stokes' theorem.Sequences and series of functions; pointwise and uniform convergence, Weierstrass M-test, differentiation and integration of series; power series, step functions and their integrals, integration of limits of increasing sequences of step functions, the Lebesgue integral and its basic properties; sets of measure zero, the monotone and dominated convergence theorems, Fatou's lemma, functions defined by integrals and differentiation under the integral sign, Fubini's theorem, square-integrable functions; completeness of* L^2*; Hilbert space axioms, the Hilbert space* l^2*; Fourier series as an isometry of* L^2 *with* l^2*; self-duality of Hilbert spaces, the Fourier series of a function, Parseval's formula; the Riesz-Fischer theorem, the* L^2*-density of trigonometric polynomials, Riemann-Lebesgue lemma, pointwise convergence of Fourier series, the Fourier transform and its properties, the Fourier integral theorem, convolution and the Fourier transform, the Laplace transform, applications to differential equations, further topics, e.g., the Dirac delta function and its Fourier transform (time permitting).*

HELLENIC MILITARY ACADEMY (1999 – present)

DURATION	COURSE	COURSE DESCRIPTION
1999-2002 (*Lecturerin Mathematics*) and 2005-2010	Differential Equations and Mathematical Theories of War	*Ordinary differential equations of first order (: homogeneous and non-homogeneous linear differential equations, differential equations with separable variables, homogeneous differential equations, exact differential equations, differential equations Bernoulli, Ricatti, Clairaut and Lagrange), numerical methods for solving differential equations (Euler, method of Taylor series with three terms,*

(*Assistant Professor/ Associate Professor/ Full Professor*)		*methods of Runge-Kutta), practical concepts of numerical solution of differential equations, A-stable Methods Runge-Kutta, pitch control, improve accuracy, methods consecutive steps (difference equations, theoretical concepts, connection method, theoretical stability method, A-stable methods consecutive steps, convergence method, class method, starting methods consecutive steps) Methods Adams, prediction-correction methods (A-stable methods of prediction-correction), recursive method, differentiation (description of method study order, stability class of methods 1 and 2, with an unchanged policy stability, stability with variable order and variable step), linear equations of first and n-th order, special types of second order differential equations (: linear homogeneous and non-homogeneous with constant or polynomial coefficients), transforms Laplace (: solving the non-homogeneous linear differential equations of higher order, functions, Heaviside, functions Dirac, generalized functions, integral equations, equations of Volterra and Fredholm), systems of linear differential equations (: general theory and methods for solving homogeneous and non- homogeneous systems), Qualitative theory of differential equations (: stability of linear systems and solutions equilibrium level phase, qualitative properties of trajectories theorem Poincaré-Bendixson), Richardson's theory of conflict and Lanchester's deterministic combat models*
2007-present (*Assistant Professor/ Associate Professor/ Full Professor*)	**Operations Research and Military Applications**	*Constrained optimization, unconstrained optimization, nonlinear programming, linear programming, graphical solving, algorithm Simplex, Tableau Simplex, Karmarcar's Algorithm, interior point methods, convex programming, bomb fragmentation, military examples of linear programming, layered defense,antiballistic missile defense and game theory: ixed strategies, optimal routes penetration function through air defense, defensive tactics, target detection, probability of penetration, tactical engagements between heterogeneously armed forces, aggregation forces, superiority parameters, optimal positioning of defence forces, best mobile defense, Markov attrition rates, deterministic models ORSBM and ISAAC, deterministic linear law, deterministic square law, stochastic and renewal models of combat casualties, probability distribution of the fighting size of a military force, combat's Chapman-Kolmogorov equations, combat's Kolmogorov Forward stochastic differential equations on corresponding transition, linear stochastic processes of losses and reinforcements in case where only a portion of reinforcing units has difficult access in the combat, stochastic differential equations ruling non-homogeneous processes of losses and reinforcements with diffcult access in the combat, destruction probability for all units of a military force at any moment of the combat, combat's Kolmogorov differential equations in the general case of a non-homogeneous stochastic process of losses and reinforcements*

		with diffcult access in the combat, limit theorems, Bonder-Farrell model
2008-present (*Assistant Professor/ Associate Professor/ Full Professor*)	Missile Allocation Strategies	*Strategies offense-last-move and defense-last-move strategies for a group of identical targets, strategies for a group of identical targets when neither side knows the other's allocation, nonpreallocation strategies for a group of identical targets, group preferential defense strategies for a group of identical targets, defense damage assessment strategies for a group of identical targets, attacker-oriented defense strategies for a group of identical targets, offensive damage assessment strategies for a group of identical targets, offense allocation to a group of targets with different values in the no-defense case, general techniques for one-sided allocation problems for a group of targets with different values, general methods for constructing two-sided offense-last-move strategies for a group of targets with different values, two-sided offense-last-move strategies for a group of targets with different values using payoff functions, two-sided offense-last-move strategies for reliable missiles, offense and defense preallocation strategies for a group of targets with different values when neither side knows the other's allocation, defense strategies for a group of targets with different values when the offensive stockpile size is unknown, attacker-oriented defense strategies for a group of targets with different values, attacks on the defense systems (models involving reliable missiles and soft radars, models involving radars resistant to damage, attacks on defensive missile silos, attacks on command and control systems), mixtures of local and area defense missiles, models for local and area missiles involving costs* and *offense and defense strategies for targets of unequal value.*

2008-present (*Assistant Professor/ Associate Professor/ Full Professor*)	Target Coverage and Target Analysis	*Survival/destruction probabilities for one or more point targets, expected fractional damage of a uniform-valued circular target, expected fractional damage of a Gaussian target, the diffused Gaussian damage fynction, attack dispersion to an area target, estimating probability of survival/destruction from impact-point data, offensive shoot-adjust-shoot strategies, attack evaluation by defense using radar information, defense strategies againt weapons of unknown lethal radius, defense strategies against a sequential attack of unknown size, defense strategies against a sequential attack by weapons of unknown lethal radius, defense strategies against a sequential attack containg exactly one weapon mixed with decoys, shoot-look-shoot defense strategies.*
1/09/2012-present (*Full Professor*)	Chaos and Complexity with Applications to the Artificial War	*What is a dynamical system, what is chaos ,what is a simulation, stability, bifurcations, more chaos, intermittency, more ergodicity, attractors, some more chaotic systems, mixing, attractor reconstruction and nonlinear prediction, symbolic dynamics, symbolic analysis of experimental data,*

		information theory, randomness and determinism, self-organization, cellular automata, heavy-tailed distributions, estimation, inference in general, active nonlinear tests of complex simulation models, inference from simulations, complex networks, agent-based models.